// NO.22-016 | 05/2022

DISCUSSION PAPER

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Entry in First-Price Auctions With Signaling

Leibniz Association

ZEW

Entry in First-price Auctions with Signaling*

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April 14, 2022

Abstract

We study the optimal entry fee in a symmetric private value first-price auction with signaling, in which the participation decisions and the auction outcome are used by an outside observer to infer the bidders' types. We show that this auction has a unique fully separating equilibrium bidding function. When the bidders' sensibility for the signaling concern is sufficiently strong, the expected revenue maximizing entry fee is the maximal fee that guarantees full participation. The larger is the bidder's sensibility, the higher is the optimal participation.

JEL: D44; D82

Keywords: First-price auction, entry, monotonic signaling; social status.

1 Introduction

In many auction settings, participants care about the information that their performance in the auction discloses to others, e.g., to other market parties, to the media or to the general public, and therefore about how their behavior is perceived by others. Giovannoni and Makris (2014) study how the outcome of a take-over auction can serve as a signal of management quality, in function of a post-auction job market for managers. Goeree (2003) shows how the outcome of a single license technology auction can reveal information to competitors about the importance of the new technology's cost reduction

*We are grateful to two anonymous referees for helpful comments and suggestions. We also thank Vincent Vannetelbosch for helpful comments.

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Olivier Bos gratefully acknowledges financial support from the ANR SIGNAL (ANR-19-CE26-0009). This research has been conducted as part of the Labex MME-DII (ANR11-LBX-0023-01) project.

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Tom Truyts gratefully aknowledges financial support from the Belgian French speaking community ARC project n°15/20-072, "Social and Economic Network Formation under Limited Farsightedness: Theory and Applications", Université Saint-Louis - Bruxelles, October 2015 - September 2020.

for the auction's winner, in function of a post-auction Cournot game.[1] Bos and Truyts (2021) consider charity and art auctions in which bidders care about how the general public perceives their altruism or wealth. In all these examples, an outside observer uses the auction's outcome to infer the private information of the bidders, and the bidders strategically adapt their bidding strategies in function of the additional signaling game implied in these auctions. Giovannoni and Makris (2014) show that, in the presence of signaling concerns, the auction's expected revenue depends on the information that the auctioneer shares with the outside observer. They investigate sealed bid auctions with different disclosure rules, i.e., in which the auctioneer releases different combinations of information to the outside observer with respect to the winning bidder's identity, a set of bids and the identities of the bidders submitting those bids. The set of revealed bids can be empty (no bids revealed), the highest bid, the second highest bid or all bids. Bos and Truyts (2021) study the second-price auction and the English auction with independent private values, in which the outside observer sees the identity and payment of the winning bidder.[2] They obtain a strict ranking of expected revenue: the second-price auction dominates the English auction. Remark that the information revealed to the outside observer in the first-price auction is the same in Bos and Truyts (2021) and Giovannoni and Makris (2014). Therefore the equilibrium bidding strategy in the first-price auction determined by Giovannoni and Makris (2014) applies also in the setting of Bos and Truyts (2021).

Entry is considered exogenously given in the above papers, but potentially gives bidders an additional signaling instrument to distinguish themselves from worse types in the context of auctions with signaling. In the present paper, we study endogenous entry in symmetric private value first-price auction with signaling. More specifically, we assume that the outside observer observes all bidders' payments and the winner's identity, i.e., observes the entry fee paid by each participating bidder, the winner's identity and the winner's payment, and uses this information to update his beliefs about the type of each bidder. The auctioneer can exploit the availability to bidders of this additional signaling instrument in order to raise additional revenue.

Public procurement is one example of an auction in which bidders have signaling concerns and in which an outsider observes the entry decision.[3] Bidders want to be selected for future tenders (Wan and Beil, 2009), and are concerned that their bidding performance can signal their intrinsic quality to others (potential business partners, future private and public procurement). The potential bidders' entry decision, the winner's identity and the price paid are public information, available on the government website. Fundraising ac-

[1]Other analyses of auctions with signaling in function of an aftermarket in industrial organization applications include Das Varma (2003) and Katzman and Rhodes-Kropf (2008).

[2]Note that this means that unlike in Giovannoni and Makris (2014), the outside observer does not observe the identity of the second-highest bidder in the second-price auction.

[3]A procurement auction is first-price *reverse* auction, i.e., in which sellers are competing in bids.

tivities is another example. Every participant pays a fee to be admitted to a fundraising event, such as a dinner, while at the same event, the participant with the highest bid wins the auction. These fundraising events often attract considerable attention on traditional and social media, not in the least by participants posting evidence of their participation online, and the outsider observes who pays during the dinner. If signaling is part of the motivation for taking part in fundraising events, then communicating who participates in the event or encouraging participants to do so can be instrumental to boost the event's expected revenue.

In this paper we study an independent private value first-price auction with entry and linear payoff functions, in a setting similar to Bos and Truyts (2021).[4] We assume that the bidders care about three things. The first two are standard: their payment and the prize if they win. In addition, the bidders care about the expected value of the outside observer's beliefs about their type. This assumption reflects many real-world examples.[5] In many art auctions and charity auctions that attract publicity (i.e., where signaling likely matters), the auction outcome is published in the press and the websites of the big auction houses and charity organizations.[6] In the European Union a directive on public procurement specified the information to be included in a contract award notice: the final price is public but all information on bids is concealed.[7]

We characterize the fully separating bidding equilibrium, and show that, when the bidders' sensibility for signaling concern is strong enough, the expected revenue maximizing entry fee is the maximal fee that guarantees full participation. In this case, the auctioneer does not set an entry fee that would allow a strict subset of bidders to distinguish themselves from worse types, but rather uses the fee to ensure that the outside observer holds, in equilibrium, the worst possible beliefs concerning a non-participating bidder. This maximal punishment for non-entry in terms of the outside observer's inferences allows the auctioneer to extract a sizable entry fee from all bidders with probability one, which maximizes the auction's expected revenue.

Entry fees are commonly used and analyzed as instruments to improve the revenue

[4]It would also be interesting to investigate other auction designs, such as the second-price auction and the English auction. However, we establish in Appendix A.9 that in the simple case of uniform distribution these two formats lead to the non-existence of a fully separating equilibrium. The bidding function in the second-price auction is also undefined for some cut off types, and this impedes the characterization of an optimal entry fee.

[5]While the examples provided do not focus the endogenous entry, they motivate the relevance of disclosure policy analyzed.

[6]For example the *Hospice de Beaune*, in Burgundy (France), uses once per year two kinds of auctions to sell wine for charity: either they reveal the price paid, or the winner's identity in addition. In the latter, the price increases by up to 500%.

[7]See Articles 21 and 22 as well as Annex V Part D of Directive 2014/24/EU of the European Parliament and of the Council of 26 February 2014 on public procurement: https://eur-lex.europa.eu/legal-content/EN/TXT/PDF/?uri=CELEX:02014L0024-20200101&from=EN (last accessed June 1st 2021).

performance of auctions. Levin and Smith (1994) show that positive entry fees maximize the expected revenue in every mechanism. More recently, Janssen et al. (2011) investigated a two-step auction game: first, bidders choose a publicly announced individual entry fee, and next, each bidder participates in the auction. Interestingly, this two-step auction in which bidders signal by means of the individual entry fees restores efficiency, despite negative externalities. Without signaling, entry fees and reserve prices are equivalent instruments, and both can be used to design an optimal auction. Reserve prices have been analyzed as signaling device by Cai et al. (2007) in the second-price auction with affiliated values, and by Tsuchihashi (2020) in the first-price auction with independent private values. However these papers study a different question: the reserve price is a signaling device used by the seller to disclose some information to the bidders. We examine how disclosure the auction's outcome to a third party affects the bidders' behavior if the latter care about the third party's beliefs about their type.

Besides the literature on signaling in auctions and auctions in general, the present paper also contributes to a growing literature about how status or signaling concerns co-determine human behavior in a variety of social and economic contexts. Cole et al. (1992) show that preferences for status can be instrumental where heterogeneous agents need to cooperate in a setting of free partner choice (employment, trade, marriage...). If the success of such a cooperation depends on the quality of the partners involved, then appearing an attractive partner is essential factor of success in a context of free partner choice, because it allows agents to high quality partners and end up in successful collaborations. Sexual selection theory also shows that the same dynamic can explain an innate taste for status is sexual species, as genes that are more successful in convincing sexual partners of their qualities tend to mate more and thus produce more offspring (see, e.g., Miller (2000)). This explains why humans tend to display an intrinsic taste for status in a variety of economic environments, including auctions. A large body of economic literature has investigated the implications of preferences for status or signaling in other situations, such as charitable behavior (e.g., Glazer and Konrad (1996), Bénabou and Tirole (2006), Ariely et al. (2009) and Kumru and Vesterlund (2010)), buying art (Mandel, 2009), conspicuous consumption (e.g., Bagwell and Bernheim (1996), Corneo and Jeanne (1997) or Hopkins and Kornienko (2004)) and economic growth (e.g., Fershtman et al. (1996), Hopkins and Kornienko (2006)).[8]

The paper is organized as follows. Section 2 introduces the formal setting. Section 3 characterizes the equilibrium and presents the main results. Finally, in Section 4 we investigate other information disclosure policies, and briefly discuss other auction formats with endogenous entry and signaling. All proofs are collected in Appendix.

[8]See Truyts (2010) for a survey of this literature.

2 Formal Setting

Consider n bidders, indexed i, bidding for a single object allocated to the highest bidder through a first-price auction. Bidder i's valuation for the object (her 'type'), is denoted V_i, and is assumed i.i.d. and drawn according to a distribution function F with support on $[\underline{v}, \overline{v}] \subset \mathbb{R}_+$. Let $f \equiv F'$ denote the density function. Bidder i's realization of V_i, denoted v_i, is her private information, but the number of bidders and the distribution F are common knowledge.

To participate in the auction, a bidder pays an entry fee $\varphi \in \mathbb{R}_+$, chosen by the auctioneer, and submits a non-negative bid. Because all bidders share the same beliefs about other bidders' valuations, they are assumed to follow a symmetric entry and bidding strategy. The entry strategy is denoted $e : [\underline{v}, \overline{v}] \to \{0, 1\}$, with $e(v) = 1$ indicating that a v type bidder pays the fee φ to participate in the auction, and the bidding strategy is denoted $\beta : [\underline{v}, \overline{v}] \to \mathbb{R}_+$. Finally, let $\boldsymbol{e}$ be the vector of entry decisions and let $\boldsymbol{b} = \boldsymbol{\beta}(\boldsymbol{v})$ denote the vector of bids given a vector of valuations $\boldsymbol{v}$, with b_i the effective bid of $i-$th bidder.

The first-price auction maps a pair of vectors describing the entry-decisions and the bids $\boldsymbol{b}$ to a winner, denoted $i^\star$, and her payment, which is her bid $b_{i^\star}$.

Apart from the auction's outcome, bidders also care about the beliefs that an uninformed party, the 'receiver', has about their type. As discussed in Bos and Truyts (2021), the receiver can be the general audience or press, a potential business partner or competitor, acquaintances of the bidder, or experts related to the object sale. The receiver is assumed to observe the entry decisions of each bidder, the auction's winner and the winner's payment $(\boldsymbol{e}, i^\star, b_{i^\star})$. The receiver's beliefs, denoted μ, are a probability distribution over the type space, such that $\mu_i \left(v | \left(\boldsymbol{e}, i^\star, b_{i^\star}\right)\right)$ is the probability of bidder i being of type v given $(\boldsymbol{e}, i^\star, b_{i^\star})$. Let $\mu \left(\boldsymbol{v} | \left(\boldsymbol{e}, i^\star, b_{i^\star}\right)\right)$ then be a probability distribution over vectors of valuations $\boldsymbol{v}$ given $(\boldsymbol{e}, i^\star, b_{i^\star})$. The receiver's beliefs are (Bayesian) consistent with an entry strategy e and a bidding strategy β if

$$\mu \left(\boldsymbol{v} | \left(\boldsymbol{e}, i^\star, b_{i^\star}\right)\right) = \frac{\Pr \left(\boldsymbol{e}, i^\star, b_{i^\star} | \boldsymbol{e}\left(v\right), \boldsymbol{\beta}\left(\boldsymbol{v}\right)\right) \prod_i f\left(v_i\right)}{\displaystyle\int \Pr \left(\boldsymbol{e}, i^\star, b_{i^\star} | \boldsymbol{e}\left(v'\right), \boldsymbol{\beta}\left(\boldsymbol{v'}\right)\right) \prod_i f\left(v'_i\right) \boldsymbol{dv'}}. \tag{1}$$

The utility of bidder i, given an auction outcome $(i^\star, b_{i^\star})$, consists of two parts. The first part is standard: the value for the object for the winner of the auction, minus the payment, consisting of the entry fee and, for the winner, the payment of his own bid. The second part is the expected value of the receiver's beliefs about bidder i's type given $(\boldsymbol{e}, i^\star, b_{i^\star})$, denoted $\mathbb{E}\left(V_i | \mu_i \left(V_i | \boldsymbol{e}, i^\star, b_{i^\star}\right)\right)$. The bidder i's sensibility for the receiver's beliefs about her type is measured by the parameter $\lambda \in (0, 1]$:[9]

[9] $\lambda = 0$ is the standard first-price auction without signaling. If $\lambda = 1$, bidder i is as sensitive for the

$$u_i(v_i, b_i|\mu_i) = \begin{cases} v_i - b_i - \varphi + \lambda \mathbb{E}\left(V_i|\boldsymbol{e}, i^\star, b_{i^\star}\right) & \text{for winner } i = i^\star \\ -\varphi + \lambda \mathbb{E}\left(V_i|\boldsymbol{e}, i^\star, b_{i^\star}\right) & \text{for participating loser } i \neq i^\star \\ \lambda \mathbb{E}\left(V_i|\boldsymbol{e}, i^\star, b_{i^\star}\right) & \text{for non-participating loser } i \neq i^\star \end{cases}$$

As in Bos and Truyts (2021), this utility function either represents a psychological game, in which bidders care directly about the receiver's beliefs, as humans care about the good opinion of others, or it is a reduced form of a game in which the receiver chooses an action given her beliefs, while the bidders care about this action.
We study the symmetric perfect Bayesian equilibria (PBE) of this auction game with signaling. A PBE is then described by a pair of strategies and beliefs (e, β, μ) such that:

1. The entry and bidding strategies (e, β) maximize the expected utility for all types v, given that all other bidders play (e, β) and given the receiver's beliefs μ.

2. The receiver's beliefs μ are Bayesian consistent with the strategies (e, β), as in (1).

As in Giovannoni and Makris (2014) and Bos and Truyts (2021), we apply the D1 criterion of Banks and Sobel (1987), which refines the set of equilibria by restricting out-of-equilibrium beliefs, in order to avoid the usual equilibrium multiplicity of signaling games. The D1 criterion restricts out-of-equilibrium beliefs by considering which bidder types are more likely to gain from an out-of-equilibrium bid, compared to their equilibrium expected utility. More precisely, if the set of beliefs for which a bidder gains from a deviation to an out-of equilibrium bid b (w.r.t. her equilibrium expected utility) is larger for one bidder type than for another, then the D1 criterion requires out-of-equilibrium beliefs to attribute zero probability to the latter type having deviated to b. In the present context, the D1 criterion imposes a certain monotonicity on out-of-equilibrium beliefs: if a certain bidder type v makes a certain bid, then a strictly higher out-of-equilibrium bid should not be attributed to a bidder type lower than v, and if no bidder type pays the entry fee in equilibrium, then a bidder who deviates to paying the entry fee should be interpreted as the highest bidder type.

3 Equilibrium Analysis

We focus on symmetric perfect Bayesian equilibria with a strictly increasing bidding function, and in which the entry decision is monotonic w.r.t. types in the sense that

outcome of the auction as for the receiver's beliefs. Therefore we consider every economic situation in which bidders give at least as much weight to the auction's material outcome as to the receiver's beliefs. Because the single crossing property of the bidder's payoff function stems from the classical part of the payoff function, i.e., the prize and payment, $\lambda \leq 1$ is required to ensure that the bidding strategy is a strictly increasing function (see the proof of Proposition 1).

there exists at most one cut off type, denoted $\tau \in [\underline{v}, \overline{v}]$, such that all bidder types with a valuation above τ choose to pay the entry fee φ in equilibrium, and that all the bidders with a valuation below τ prefer to stay out.

As discussed above we restrict out-of-equilibrium beliefs by means of the D1 criterion. Although the D1 criterion typically excludes (semi)pooling PBE in monotonic signaling games at the one hand, and although (semi-)pooling strategies are normally easily excluded in auctions with the present preference structure at the other hand, the exercise of excluding (semi-)pooling equilibria by means of the D1 criterion is less obvious when both games are combined into an auction with signaling concerns. The reason is that bidders cannot be excluded to bid above their valuation for the object (and typically do so in equilibrium). As usual, the D1 criterion ensures that the receiver puts zero probability on all types lower than the maximal type in a pool when observing a bid marginally above the common bid in this pool. In monotonic signaling games this implies that a marginal increase above the pool's signal is rewarded by a discrete jump in terms of inference by the receiver, which immediately excludes (semi-)pooling equilibria. In the present setting, however, and as in Giovannoni and Makris (2014) and Bos and Truyts (2021), such a marginal increase in bid also increases a deviating bidder's chances of winning the auction, and thereby her expected payment, by a discrete amount.

Lemma 1. *If $f'(.) \leq 0$, all first-price auction D1 PBE are fully separating, with $\beta'(v) > 0$ for all $v \in [\tau, \overline{v}]$ and $\tau \in [\underline{v}, \overline{v}]$.*

The condition $f'(.) \leq 0$ is (amply) sufficient to ensure that the D1 criterion has enough bite to exclude the many semi-pooling and polling in the D1 PBE. Although restrictive, this condition is likely satisfied if we believe that only the top end of e.g. the income distribution participates in a public procurement or in lobbying activities. More importantly, however, in the analysis with no entry fee, Giovannoni and Makris (2014) establish the log-concavity of F^{n-1} is a sufficient condition for the first-price auction, while Bos and Truyts (2021) show f strictly decreasing is close to a necessary condition for the existence of a fully separating PBE in both the second-price and English auctions. Despite their analysis is without entry fee, that makes it relevant for comparability. Note that this condition implies the common log-concavity of F or the non-decreasing hazard rate condition, but is neither weaker nor stronger than the log-concave density condition imposed by Goeree (2003). A similar condition is also available in Segev and Sela (2014).

Let us then consider the problem of a type v bidder who wishes to pay the entry fee φ in order to participate in the auction. If the PBE is fully separating, then the type of the bidder who wins the first price auction is fully revealed to be $\beta^{-1}(\beta(v_{i^\star})) = v_{i^\star}$ in equilibrium. If the auction's winner is of type $v_{i^\star}$ and if only the bidders with a valuation above $\tau \leq v_{i^\star}$ decide to participate, then all losing participating bidders are estimated to be of type $\frac{\int_{\tau}^{v_{i^\star}} xdF(x)}{F(v_{i^\star}) - F(\tau)}$. However, in the contingency that the type v bidder does not win the auction, he *ex ante* does not know the type of the winner, except that the winner must have a higher valuation than his. Therefore, the type v bidder takes the expectation over the winning bidder's type, conditional on the fact that it is higher than his. As such, the expected value of the receiver's beliefs about the v type bidder in case of losing the auction and an entry cut off type τ is

$$\frac{1}{1-F^{n-1}(v)} \int_{v}^{\overline{v}} \frac{\int_{\tau}^{y} xdF(x)}{F(y)-F(\tau)} dF^{n-1}(y).$$

Finally, if the bidding function is strictly increasing, then the v bidder's probability of winning the auction is equal to the probability of the $n-1$ other bidders having a valuation lower than v, i.e., $F^{n-1}(v)$.

Bringing all this together, we consider the problem of a v type bidder who decides to enter the auction, and seeks to maximize his expected payoff. Following a common mechanism design practice, we understand this problem of the v type bidder as a problem of choosing another type $\tilde{v}$, whose equilibrium strategy the type v wants to imitate and probability of winning and expected inferences by the receiver he wants to obtain, in order to maximize his expected payoff. Thus, given an equilibrium bidding function β, the problem of a v type bidder is:

$$\max_{\tilde{v}} \left\{ F^{n-1}(\tilde{v}) \left[v - \beta(\tilde{v}) + \lambda \tilde{v} \right] + \lambda \int_{\tilde{v}}^{\overline{v}} \frac{1}{F(y)-F(\tau)} \int_{\tau}^{y} xdF(x) dF^{n-1}(y) - \varphi \right\}. \quad (2)$$

The first order condition is

$$\left(F^{n-1}(\tilde{v})\beta(\tilde{v})\right)' = (F^{n-1}(\tilde{v}))'(v+\lambda\tilde{v}) + \lambda F^{n-1}(\tilde{v}) - \frac{\lambda}{F(\tilde{v})-F(\tau)} \int_{\tau}^{\tilde{v}} xdF(x)(F^{n-1}(\tilde{v}))'. \quad (3)$$

Of course, in equilibrium the bidding function must be such that each bidder type strictly prefers his own equilibrium bid to imitating another type, such that we impose $\tilde{v} = v$.

Proposition 1. *If $f'(.) \leq 0$, then for a given entry fee φ and cut off type τ a unique D1 PBE exists, and its bidding strategy is*

$$\beta(v) = \lambda v - \lambda \frac{F^{n-1}(\tau)}{F^{n-1}(v)} \tau + \frac{1}{F^{n-1}(v)} \int_{\tau}^{v} \left(y - \frac{\lambda \int_{\tau}^{y} xdF(x)}{F(y)-F(\tau)} \right) dF^{n-1}(y), \quad (4)$$

such that $\beta(\tau) = 0$ and $\beta'(v) > 0$ for all $v \in (\tau, \overline{v})$.

The proof of Proposition 1 first establishes that in equilibrium, it must be that $\beta(\tau) = 0$, because the τ type bidders otherwise have a strict incentive to deviate to a zero bid.

The proof then derives the equilibrium bidding function and finally demonstrates that this equilibrium satisfies the necessary global strict second order conditions.

Note that the bidding strategy described by (4) evaluated at $\tau = \underline{v}$ corresponds to the bidding function without entry fee in Giovannoni and Makris (2014). The following Corollary shows that an entry fee, and the related entry decision, leads to underbidding at the equilibrium.

Corollary 1. *The payment of an entry fee induces bidders to bid less aggressively at the equilibrium. Moreover, bidding strategies are decreasing with the cut off* τ.

The entry decision affects the equilibrium in two opposite ways. First, compared to the equilibrium with exogenous participation, the presence of entry fee makes that participating losers are inferred as higher types, which is the expected average over $[\tau, v_{i^\star}]$ instead of $[\underline{v}, v_{i^\star}]$ (with $v_{i^\star}$ the winner type) than losers in an auction with exogenous participation. Second, bidders internalize the entry fee by deducting it from their equilibrium bids, which implies lower equilibrium bidding than in the case with exogenous participation.

We now turn to the bidders' entry decisions. For entry fees that induce participation by only a strict subset of the type space, the bidder type with cut off valuation τ must be indifferent between paying the entry fee to participate in the auction on one hand, and staying out on the other hand. If the cut off type τ stays out, he pools with all the non-participating lower bidders and obtains a payoff from the receiver's inferences equal to $\lambda \frac{\int_{\underline{v}}^{\tau} v dF(v)}{F(\tau)}$. If the τ type decides to pay the entry fee, he wins the auction with probability $F^{n-1}(\tau)$ with a zero bid, in which case he obtains the object he values τ and is perceived as type τ by the receiver. Otherwise, he gets the expected inference of a losing participating bidder

$$\lambda \int_{\tau}^{\overline{v}} \frac{\int_{\tau}^{y} x dF(x)}{F(y) - F(\tau)} dF^{n-1}(y).$$

Therefore, the equilibrium entry strategies for an internal $\tau \in (\underline{v}, \overline{v})$ are characterized by the following relationship between the entry fee φ and the cut off type τ:

$$\varphi = F^{n-1}(\tau)(1+\lambda)\tau + \lambda \int_{\tau}^{\overline{v}} \frac{\int_{\tau}^{y} x dF(x)}{F(y) - F(\tau)} dF^{n-1}(y) - \lambda \frac{\int_{\underline{v}}^{\tau} y dF(y)}{F(\tau)}. \tag{5}$$

Hence, the maximal entry fee φ that the cut off type τ is willing to pay is equal to the sum of the expected prize and the difference between the receiver's expected inferences about a participating bidder and a nonparticipating bidder. A quick inspection of (5) shows us that, first, the maximal entry fee guaranteeing full participation is

$$\hat{\varphi} = \lambda \left(\int_{\underline{v}}^{\overline{v}} y - \frac{\int_{\underline{v}}^{y} F(x)dx}{F(y)} dF^{n-1}(y) - \underline{v} \right),$$

second, the lowest fee guaranteeing no participation is

$$\bar{\varphi} = (1 + \lambda)\overline{v} - \lambda \mathbb{E}(V),$$

i.e., the sum of the inference and prize the $\overline{v}$ type bidder gets with probability 1 if he participates minus what he gets if he pools with all the other non-participating bidders, and, third, φ strictly increases with τ in the interval $[\hat{\varphi}, \bar{\varphi}]$.

This characterization of the equilibrium bidding and entry decisions now allows us to proceed to the final step: what entry fee should the auctioneer choose in order to maximize the auction's expected revenue? The expected revenue of the auction consists of both the expected entry fees paid by the participating bidders and the winner's expected payment:

$$\mathbb{E}R(\tau) = n\varphi\,(1 - F(\tau)) + \int_{\tau}^{\overline{v}} \beta(v) dF^{n-1}(v).$$

Increasing the entry fee beyond $\hat{\varphi}$ means that the auctioneer collects a higher entry fee from the participating bidders, increases the risk that bidders choose to stay out and decreases the equilibrium bid of all participating bidders. The following Proposition characterizes the optimal entry fee.

Proposition 2. *In the fully separating D1 PBE:*

(*i*) *If the bidders' sensitivity for signaling concerns λ is sufficiently large, i.e., sufficiently close to 1, then the expected revenue is maximal at $\hat{\varphi}$, i.e., the maximal entry fee that guarantees full participation.*

(*ii*) *The maximal entry fee that guarantees full participation, $\hat{\varphi}$, increases with λ.*

In order to maximize the auction's expected revenue, the auctioneer chooses φ such that the receiver holds the worst possible beliefs about a non-participating bidder, i.e., that the bidder is of type $\underline{v}$ with probability one. If the bidders' sensitivity for signaling concerns λ is sufficiently strong, then the auctioneer fully exploits the bidders' fear of being singled out as the worst type by not participating, in order to collect the maximal sum of entry fees from all bidders. Note that this full participation for level of λ sufficiently large contrasts with the optimal entry fee of the equivalent auction without signaling, where the optimal fee must exclude a part of the bidder types from participation. If bidders attach more importance to the receiver's beliefs, i.e., have a higher λ, then this gives the auctioneer more leverage to exploit the bidders' fear of being singled out as the worst types $\underline{v}$, allowing him to ask a higher entry fee from all bidders. The proof of Proposition 2 first demonstrates the following result, which is presented here as a Lemma, and which we show to be equivalent to stating Proposition 2.

Lemma 2. *If the bidders' sensibility to signaling λ is sufficiently large, i.e., sufficiently close to 1, then the bidders' ex ante expected payoffs strictly increase with the entry fee φ, for $\varphi \in [\hat{\varphi}, \bar{\varphi}]$.*

A bidder's *ex ante* payoff consists of his expected prize, $\int_\tau^{\overline{v}} F^{n-1}(v) v dF(v)$, the expected inferences of the receiver, and his expected payment as a negative, where the latter consists of the entry fee and the expected value of paying the winner's bid, i.e., $\int_\tau^{\overline{v}} F^{n-1}(v)\beta(v)dF(v)$.

$$\begin{aligned} EU(\tau) &= \lambda \int_{\underline{v}}^{\tau} y dF(y) + \int_{\tau}^{\overline{v}} F^{n-1}(v)\left(v(1+\lambda) - \beta(v)\right) dF(v) \\ &\quad + \lambda \int_{\tau}^{\overline{v}} \int_{v}^{\overline{v}} \frac{\int_\tau^y x dF(x)}{F(y) - F(\tau)} dF^{n-1}(y) dF(v) - (1 - F(\tau))\varphi \end{aligned}$$

The receiver's Bayesian beliefs are a martingale. Because the receiver's equilibrium beliefs are by construction consistent, the law of total expectation implies that the *ex ante* expectation of the receiver's beliefs must equal $\mathbb{E}(V)$, and be as such independent of φ. Indeed, if the receiver's equilibrium beliefs are consistent with the observed auction's outcome, and if in the bidder's problem we take the expectation of these beliefs over the other bidders' types given the bidder's own type, then taking the *ex ante* expectation of these expectations with respect to the bidder's own type must necessarily result in the prior expectation $\mathbb{E}(V)$, irrespective of the value of φ. Therefore, the *ex ante* expectation of the receiver's equilibrium beliefs is independent of φ, such that we can focus on the other components of $EU(\tau)$. The expected prize decreases with φ, because increasing φ in $[\hat{\varphi}, \bar{\varphi}]$ increases the probability that no bidder will wish to pay the entry fee, and that the object thus remains with the auctioneer. Hence, if the *ex ante* expected payoff increases with φ, the receiver's *ex ante* expected inferences are independent of φ, and the *ex ante* expected prize decreases with φ, then it must be that the *ex ante* expected payment, and thereby the auction's expected revenue, strictly decreases with φ.

We conclude this section with an illustration of our results for uniform distribution, and show that a level of $\lambda \leq 1$ is sufficient to guarantee full participation.

Example 1. Consider valuations uniformly distributed on $[0, 1]$. Using equation (11) from the proof of Lemma 2 in Appendix, it follows

$$\frac{dEU}{d\tau}(\tau) = \frac{\lambda}{2} - \tau^{n-1} + \tau^n,$$

which is always positive for $\lambda > \underbrace{\frac{2}{n}\left(\frac{n-1}{n}\right)^{n-1}}_{\leq 1}$. Furthermore if $n = 2$, for every $\lambda \in (\frac{1}{2}, 1]$ the optimal fee is equal to the maximal fee that guarantees full participation.

4 Discussion

We have shown that the optimal entry fee in an independent private value first-price auction is the maximal fee that guarantees full participation. What about other auction formats and other information disclosure policies?

4.1 Other information disclosure policies

The information disclosure policy discussed so far corresponds with the examples presented in the introduction. Yet, different information disclosure assumptions equally merit attention, since they can be equally plausible in other circumstances. Among them, we distinguish the following two interesting disclosure policies: i) the receiver observes the participation decisions and the winner's identity, and ii) the receiver sees the participation decisions and all bids (and corresponding bidders' identities). In this subsection, we investigate how endogenous entry affects the bidding strategies in the first-price auction under these two alternative information disclosure policies, first studied in Giovannoni and Makris (2014).

Participation decisions and the winner's identity revealed

Let us first study the equilibrium bidding strategies with endogenous participation if the participation decisions and only the winner's identity are disclosed to the receiver. We denote by β^{Id} and φ^{Id} the corresponding bidding function and entry fee. Without any information about either the payment or the bids, and if only the bidders with a valuation above τ decide to participate, the receiver uses the information about who is the winner to form his beliefs about the different bidders. Therefore the *ex ante* expectation of the receiver's beliefs about the winner's type is the expected value of highest draw out of n, $\int_{\tau}^{\bar{v}} \mathbb{E}(V|V \geq y)dF^{n-1}(y)$. The receiver cannot distinguish between the participating losers, and so his expectation of a loser's type is the expected type of a bidder with valuation between the winner's type and τ, i.e., $\int_{\tau}^{\bar{v}} \mathbb{E}(V|V \leq y)dF^{n-1}(y)$.

Therefore, the expected utility of a valuation v bidder, who decides to enter the auction, choosing type $\tilde{v}$'s bidding strategy is:

$$F^{n-1}(\tilde{v})(v-\beta^{Id}(\tilde{v})+F^{n-1}(\tilde{v})\lambda\int_{\tau}^{\bar{v}}\mathbb{E}(V|V\geq y)dF^{n-1}(y)+(1-F^{n-1}(\tilde{v}))\lambda\int_{\tau}^{\bar{v}}\mathbb{E}(V|V\leq y)dF^{n-1}(y)-\varphi^{Id} \tag{6}$$

Note that the winning and losing expected inferences are both independent of $\tilde{v}$, as in the case without endogenous entry in Giovannoni and Makris (2014). Note also that in this information disclosure policy, we face no equilibrium selection issues with respect to the bidding strategies. Therefore the equilibrium bidding function resembles the one of an auction without signaling and an entry fee, but with an additional markup reflecting the difference between the receiver's expected inference of a winner's and loser's type as an additional prize to winning. The following Proposition characterizes the bidding equilibrium and the equilibrium entry strategies.

Proposition 3. *For a given entry fee φ^{Id} and corresponding cut off type τ, the fully separating PBE bidding strategy is*

$$\beta^{Id}(v)=\frac{1}{F^{n-1}(v)}\int_{\tau}^{v}ydF^{n-1}(y)+\lambda\left(\int_{\tau}^{\bar{v}}\mathbb{E}(V|V\geq y)dF^{n-1}(y)-\int_{\tau}^{\bar{v}}\mathbb{E}(V|V\leq y)dF^{n-1}(y)\right),$$

such that $\beta^{Id}(\tau)=0$, for all $v\in(\tau,\bar{v})$.

The equilibrium entry strategies for an internal $\tau\in(\underline{v},\bar{v})$ are characterized by the following relationship between the entry fee φ^{Id} and the cut off type τ:

$$\varphi^{Id}=F^{n-1}(\tau)\tau+\lambda\int_{\tau}^{\bar{v}}\mathbb{E}(V|V\geq y)dF^{n-1}(y)+\lambda\int_{\tau}^{\bar{v}}\mathbb{E}(V|V\leq y)dF^{n-1}(y)-\lambda\frac{\int_{\underline{v}}^{\tau}ydF(y)}{F(\tau)}. \tag{7}$$

It follows from equation (7) that the maximal entry fee guaranteeing full participation is

$$\hat{\varphi}^{Id}=\lambda\int_{\underline{v}}^{\bar{v}}\mathbb{E}(V|V\geq y)dF^{n-1}(y)+\lambda\int_{\underline{v}}^{\bar{v}}\mathbb{E}(V|V\leq y)dF^{n-1}(y),$$

and the lowest fee guaranteeing no participation is $\bar{\varphi}^{Id}=\bar{v}-\lambda\mathbb{E}(V)$.

As in the case where the winner's identity and payment are revealed, the bidding strategy evaluated at the lowest value of $\tau=\underline{v}$ corresponds to the bidding strategy without entry fee, presented in Giovannoni and Makris (2014). A quick inspection of the bidding function reveals the impact of endogenous entry on the bidding strategies.

Corollary 2. *Endogenous entry makes participating bidders bid less aggressively at the equilibrium. Moreover, bidding strategies are decreasing with the cut off τ.*

As before, bidders internalize the cost of the entry fee when determining their optimal bid, such that the entry fee exerts a downward pressure on the equilibrium bids.

Participation decisions and all bids (and corresponding bidders' identities) revealed

We now provide an analysis for the bidding strategies with endogenous participation when all participation decisions and all bids (and bidders' identities) are disclosed to the receiver. We denote by β^A and φ^A the corresponding bidding function and entry fee. In the case of a fully separating equilibrium (i.e., with a strictly increasing bidding function), revealing the bids allows the receiver to infer the true valuation v of each participating bidder. Therefore the receiver's expectation of all participating bidders, a loser's valuation as well a winner's valuation is their true valuation.

Therefore, the expected utility of a valuation v bidder, who decides to enter the auction, choosing type $\tilde{v}$'s bidding strategy is:

$$F^{n-1}(\tilde{v})(v - \beta^A(\tilde{v}) + \lambda\tilde{v} - \varphi^A \tag{8}$$

The following Proposition characterizes the equilibrium bidding and entry strategies. We do not elicit conditions that guarantee an essentially unique D1 fully separating equilibrium, and we limit the scope of this section to a discussion of how endogenous entry affects the bidding equilibrium when all bids and entry decisions are disclosed. Giovannoni and Makris (2014) determine the bidding equilibrium without endogenous entry for the same existence condition below (concavity of F^{n-1}), and show this equilibrium satisfies the D1 criterion.[10]

Proposition 4. *If F^{n-1} is concave, then for a given entry fee φ^A and cut off type τ, the fully separating PBE bidding strategy is*

$$\beta^A(v) = \frac{1}{F^{n-1}(v)} \int_\tau^v y dF^{n-1}(y) + \lambda \frac{v - \tau}{F^{n-1}(v)},$$

such that $\beta^A(\tau) = 0$ and $(\beta^A(v))' > 0$ for all $v \in (\tau, \overline{v})$.

The equilibrium entry strategies for an internal $\tau \in (\underline{v}, \overline{v})$ are characterized by the following relationship between the entry fee φ^A and the cut off type τ:

$$\varphi^A = F^{n-1}(\tau)\tau + \lambda\tau - \lambda \frac{\int_{\underline{v}}^{\tau} y dF(y)}{F(\tau)}. \tag{9}$$

It follows from equation (9) that the maximal entry fee guaranteeing full participation is $\hat{\varphi}^A = \lambda\underline{v}$, and that the lowest fee guaranteeing no participation is $\bar{\varphi}^A = \overline{v}(1+\lambda) - \lambda\mathbb{E}(V)$.

Note that the equilibrium bidding strategy evaluated at $\tau = \underline{v}$ corresponds to the bidding strategy without entry fee presented in Giovannoni and Makris (2014). The

[10] See Giovannoni and Makris' (2014) Proposition 5.

bidding function in Proposition 4 shows how endogenous participation affects the bidding strategies. This is summarized in the following Corollary.

Corollary 3. *Endogenous entry makes participating bidders bid less aggressively in equilibrium, compared to an equivalent auction with exogenous participation. Moreover, the equilibrium bidding strategies are decreasing with the cut off* τ*.*

Contrary to the case where only the winner's identity and her payment are disclosed, the entry fee does not affect the receiver's *ex ante* expected inference of the participating losers' types. Therefore, endogenous participation makes less of a difference for the receiver's equilibrium beliefs, and thereby for the equilibrium bidding function, compared to the equivalent auction with exogenous participation.

4.2 Other auction formats

What about entry fees in other auction formats with signaling when the winner's identity and her payment are disclosed, such as the all-pay auction, the second-price auction or the English auction? The all-pay auction constitutes a simple exercise, because endogenous entry affects the signaling incentives and expected inferences by the receiver in the same way in the first-price and all-pay auctions. As in Giovannoni and Makris (2014), the all-pay auction is equivalent to the first-price auction in terms of expected payments and expected revenues. For this reason, the characterization of the optimal entry fee in the all-pay auction is the same as in the first-price auction.

The introduction of endogenous entry in the second-price and in the minimal information English button auctions with signaling (see Bos and Truyts (2021)) proves to be more complicated. The principal reason is that if we maintain our reference information disclosure policy (the receiver only observes the winner's payment and the participation decisions), then the receiver can only infer the valuation of the second highest bidder from the winning bidder's payment. Being unable to distinguish between the losing bidders, the receiver knows that one of the losing bidders has a valuation that is reflected in the winner's payment, say v, while the other participating losing bidders have a valuation between τ and v. However, for $\tau \in (\underline{v}, \overline{v})$ the number of participating losing bidders depends on how many bidders randomly drawn a valuation above τ. While the receiver observes the actual participation decisions, the bidders need to consider all possible numbers of participating bidders when determining their bidding strategy. Therefore, their expectation of the receiver's beliefs about a losing participating bidder's type, given a second highest type v and a cut off type τ, is:

$$\frac{1}{F^{n-2}(v)} \sum_{i=0}^{n-2} \binom{n-2}{i} \frac{F^{n-2-i}(\tau)\left(F(v)-F(\tau)\right)^{i}}{i+1} \left(v + i\frac{\int_{\tau}^{v} y dF(y)}{F(v)-F(\tau)}\right). \qquad (10)$$

Of course, a participating bidder does not know *ex ante* the valuation of the second-highest bidder if this valuation turns out to be higher than his own, and he must consider the expected value of (10) w.r.t. v in order to determine his optimal bidding strategy. As a result, determining the optimal bidding strategy, and even more so the expected revenue, becomes a very tedious exercise. Moreover, the existence of a fully separating equilibrium is far from guaranteed. In Appendix A.9, we show the non-existence of a fully separating equilibrium in the English and the second-price auction for F being the uniform distribution on the unit interval, by demonstrating that the optimal bidding strategy is non-increasing for a nontrivial subset of type space. The bidding function is also undefined at τ in the second-price auction with uniform distribution, and this impedes the characterization of an optimal entry fee. These two complications impede the characterization of an optimal entry fee for the simple case of the uniform distribution.

A Appendix

A.1 Proof of Lemma 1

Following arguments in Bos and Truyts (2021), note that the formal implementation of the D1 criterion depends on the auction format. Formally, for types v', v'' and out-of-equilibrium message m, beliefs μ, a utility function $u(m, \mu|v)$ and equilibrium utility levels $u^\star(v)$, define the following two sets of beliefs which make a type v sending m resp. strictly better off than in equilibrium and equally well off as in equilibrium:

$$\mathcal{M}^+(m, v) = \{\mu | u(m, \mu|v) > u^\star(v)\}$$
$$\mathcal{M}^0(m, v) = \{\mu | u(m, \mu|v) = u^\star(v)\}.$$

Then the D1 criterion requires

$$\mathcal{M}^+(m, v') \cup \mathcal{M}^0(m, v') \subset \mathcal{M}^+(m, v'') \Longrightarrow \mu(v'|m) = 0.$$

We proceed in three steps to prove Lemma 1: 1. Establish for any D1 PBE the bidding function β is weakly increasing, 2. Show in any D1 PBE, there is no pooling with the τ type and 3. Show in any D1 PBE, there is no pooling above the τ type.

Claim 1 (β weakly increasing)**.** *In any D1 PBE, if type v' chooses b', then no $v'' < v'$ bids $b'' > b'$.*

Proof. Let $p(b)$ denote the probability of winning the auction with bid b and $E_w(b)$ and $E_l(b)$ the expected values of the receiver's inference about respectively a winning and losing bidder who bids b. Assume that type v' bids b' in equilibrium and gets expected inferences $E_w(b')$ and $E_l'(b')$. Let (E_w'', E_l'') a pair of inferences such that type v' is indifferent between bidding b'' and getting inference (E_w'', E_l'') and her equilibrium payoff, i.e.

$$p(b')(v'-b')+\lambda E_l(b')+p(b')\lambda[E_w(b')-E_l(b')]-\psi = p(b'')(v'-b'')+\lambda E_l''+p(b'')\lambda[E_w''-E_l'']-\psi$$

It follows that

$$\begin{aligned}[p(b'')-p(b')]v' &= A \\ &\equiv p(b'')b''-p(b')b'+\lambda E_l(b')+p(b')\lambda[E_w(b')-E_l(b')]-(\lambda E_l''+p(b')\lambda[E_w''-E_l''])\end{aligned}$$

Note that $p(b'')-p(b')\geq 0$. Then if $p(b'')-p(b')>0$, it must be that $[p(b'')-p(b')]v''<A$ such that,

$$p(b')(v''-b')+\lambda E_l(b')+p(b')\lambda[E_w(b')-E_l(b')] > p(b'')(v''-b'')+\lambda E_l''+p(b')\lambda[E_w''-E_l''].$$

Hence, the lower valuation type needs a higher compensation in terms of inference for a higher bid.

Assume then that the equilibrium expected utility of type v'' is low enough to make $\mathcal{M}^+(b'',v')\subseteq\mathcal{M}^+(b'',v'')\cup\mathcal{M}^0(b'',v'')$. Then it must be that the v'' strictly prefers bundle $(b',E_w(b'),E_l(b'))$ to her equilibrium strategy, a contradiction. Therefore $\mu(v''|b'')=0$ in the D1 PBE, and no v'' type with $v''<v'$ chooses a b'' bid with $b''>b'$ if type v' bids b' in equilibrium. □

Claim 2 (No pooling with τ)**.** *No type $v>\tau$ pools with type τ in the D1 PBE.*

Assume an equilibrium in which a non-degenerate set of types $\mathcal{O}=\left\{v|\beta(v)=\tilde{b}\right\}$ pool at $\tilde{b}$, such that $\tau\in\mathcal{O}$. By Claim 1, $\mathcal{O}$ is a convex set.

If $\tilde{b}>\tau$, then a type τ bidder can strictly improve herself by deviating to τ. Such deviation is never observed, such that the receiver's inference about the τ bidder is not worse, but she avoids winning the auction to pay $\tilde{b}$ in excess of her valuation τ.

If $\tilde{b}\leq\tau$, then note that the expected inference about a bidder in $\mathcal{O}$ is $E_w(\tilde{b})=\frac{1}{|\mathcal{O}|}\int_{\mathcal{O}} v\,\mathrm{d}F(v)$ and $E_l(\tilde{b})$. The probability of winning when pooling at $\tilde{b}$ is $\frac{F(\sup(\mathcal{O}))^{n-1}}{n}$. Consider then type $\sup(\mathcal{O})$. If she bids a $\tilde{b}+\varepsilon$, with $\varepsilon>0$, she wins at least with probability $F(\sup(\mathcal{O}))^{n-1}$, in which case $E_w(\tilde{b}+\varepsilon)\geq\sup(\mathcal{O})$ and $E_l(\tilde{b}+\varepsilon)>E_l(\tilde{b})$, while $\sup(\mathcal{O})-\tilde{b}-\varepsilon>0$ for ε sufficiently small. But in equilibrium it must be that

$$\begin{aligned}&\frac{F^{n-1}(\sup(\mathcal{O}))}{n}\left(\sup(\mathcal{O})-\tilde{b}+\lambda E_w(\tilde{b})\right)+\left(1-\frac{F^{n-1}(\sup(\mathcal{O}))}{n}\right)\lambda E_l(\tilde{b})\\ &\geq F^{n-1}(\sup(\mathcal{O}))\left((1+\lambda)\sup(\mathcal{O})-\tilde{b}-\varepsilon\right)+\left(1-F^{n-1}(\sup(\mathcal{O}))\right)\lambda E_l(\tilde{b}+\varepsilon),\end{aligned}$$

which is only true for $\varepsilon\to 0$ if $\sup(\mathcal{O})=\tau$ and $n=1$.

Claim 3 (No pooling above τ)**.** *In the D1 PBE, no bid $\tilde{b}$ is chosen by two types $v' \neq v''$.*

Proof. Assume a D1 PBE in which $\tilde{b}$ is the lowest bid chosen by a nondegenerate set of types $\mathcal{O} = \left\{v|\beta(v) = \tilde{b}\right\}$. Note that $\mathcal{O}$ is convex by Claim 1 and $\inf(\mathcal{O}) > \tau$ by Claim 2. By the continuity of f and of the utility function w.r.t. all arguments, the $\inf(\mathcal{O})$ must in equilibrium be indifferent between separating at $\lim_{v \to \inf(\mathcal{O})^-} \beta(v)$ and pooling at $\tilde{b}$. Note then that the indirect utility difference between $\sup(\mathcal{O})$ and $\inf(\mathcal{O})$ in the pooling equilibrium is

$$p(\tilde{b})\left(\sup(\mathcal{O}) - \inf(\mathcal{O})\right).$$

In the separating equilibrium this is by the envelope theorem

$$\int_{\inf(\mathcal{O})}^{\sup(\mathcal{O})} F^{n-1}(x)\ \mathrm{d}x = \sup(\mathcal{O})\, F^{n-1}(\sup(\mathcal{O})) - \inf(\mathcal{O})\, F^{n-1}(\inf(\mathcal{O})) - \int_{\inf(\mathcal{O})}^{\sup(\mathcal{O})} x\ \mathrm{d}F^{n-1}(x).$$

If $\sup(\mathcal{O}) = \inf(\mathcal{O})$ these are both equal to zero, but by the same differential argument as in Lemma 1 (Claim 3) of Bos and Truyts (2021),

$$p(\bar{b})\left(\sup(\mathcal{O}) - \inf(\mathcal{O})\right) < \int_{\inf(\mathcal{O})}^{\sup(\mathcal{O})} F^{n-1}(x)\ \mathrm{d}x$$

The condition $f'(.) \leq 0$ imposed to guarantee the existence of a separating equilibrium, always guarantees this inequality.

□

A.2 Proof of Proposition 1

We proceed in 3 steps: 1) demonstrating that in a D1 PBE $\beta(\tau) = 0$, 2) deriving the bidding function in Proposition 1, and 3) If $f'(.) \leq 0$, then for a given entry fee φ and cut off type τ showing that β satisfies the second order condition.

Step 1: Suppose that in equilibrium $\beta(\tau) > 0$.

Remark that because β is a strictly increasing function, the τ type bidder who pays the entry fee can only win if all other bidders have a valuation strictly smaller than τ. This happens with probability $F^{n-1}(\tau)$. If the τ type bidder pays the entry fee but deviates to a zero bid, he still wins the auction with probability $F^{n-1}(\tau)$. However in this case he pays no longer his strictly positive bid as a winner. A receiver with D1 beliefs attributes such an out-of-equilibrium bid to at least the τ bidder. Hence, this deviation constitutes a strict improvement for the bidder, such that $\beta(\tau) > 0$ is not consistent with a D1 PBE.

Step 2: From (3) after imposing $\tilde{v} = v$ and rewriting, we obtain

$$\left(F^{n-1}(v)\beta(v)\right)' = \lambda\left(F^{n-1}(v)v\right)' + \left(F^{n-1}(v)\right)' v - \lambda \frac{\int_\tau^v x dF(x)}{F(v) - F(\tau)}\left(F^{n-1}(v)\right)'$$

After integration and using $\beta(\tau)=0$, it follows

$$F^{n-1}(v)\beta(v)=\lambda F^{n-1}(v)v-\lambda F^{n-1}(\tau)\tau+\int_{\tau}^{v}ydF^{n-1}(y)-\lambda\int_{\tau}^{v}\frac{\int_{\tau}^{y}xdF(x)}{F(y)-F(\tau)}dF^{n-1}(y),$$

which, using a few elementary algebraic operations, can be rewritten into the bidding function in Proposition 1.

Step 3. We proceed in 2 steps: 1) showing that a strictly increasing bidding function implies local strict concavity of the bidder's problem, and 2) demonstrating that the equilibrium bid is then a global expected utility maximizing choice for each bidder.

First, use the first order condition (3) to denote

$$\begin{aligned}G(\tilde{v},v)\equiv{}&\left(F^{n-1}(\tilde{v})\right)'(v+\lambda\tilde{v})-\left(F^{n-1}(\tilde{v})\beta(\tilde{v})\right)'\\&+\lambda F^{n-1}(\tilde{v})-\frac{\lambda}{F(\tilde{v})-F(\tau)}\int_{\tau}^{\tilde{v}}xdF(x)\left(F^{n-1}(\tilde{v})\right)'=0,\end{aligned}$$

which defines $\beta(v)$ for $\tilde{v}=v$. Let us denote G_1 and G_2 the derivatives G with respect of its first and second arguments. By the implicit function theorem $\beta'(v)>0$ if and only if

$$-\frac{G_2(\tilde{v},v)}{G_1(\tilde{v},v)}=-\frac{\left(F^{n-1}(\tilde{v})\right)'}{G_1(\tilde{v},v)}>0,$$

which is only satisfied if $G_1(\tilde{v},v)<0$ for all v at $\tilde{v}=v$. To see that β globally maximizes the bidder's problem, note that, by construction, $G(\tilde{v},v)=0$ is satisfied at $\tilde{v}=v$, while $G_2(\tilde{v},v)>0$ for all $\tilde{v}>v$, such that type v's utility reaches a unique maximum at $\tilde{v}=v$. Hence, we have that the second order condition is satisfied if $\beta'(v)>0$ for all $v\in[\underline{v},\overline{v}]$. Note then that:

$$\beta'(v)=\lambda+\frac{(n-1)f(v)}{F(v)}\left(\begin{array}{c}\lambda\frac{F^{n-1}(\tau)}{F^{n-1}(v)}\tau+\left(v-\lambda\frac{\int_{\tau}^{v}xdF(x)}{F(v)-F(\tau)}\right)\\-\frac{1}{F^{n-1}(v)}\int_{\tau}^{v}\left(y-\lambda\frac{\int_{\tau}^{y}xdF(x)}{F(y)-F(\tau)}\right)dF^{n-1}(y)\end{array}\right)>0,$$

because

$$\begin{aligned}&\frac{1}{F^{n-1}(v)}\int_{\tau}^{v}\left(y-\lambda\frac{\int_{\tau}^{y}xdF(x)}{F(y)-F(\tau)}\right)dF^{n-1}(y)\\<\;&\frac{1}{F^{n-1}(v)-F^{n-1}(\tau)}\int_{\tau}^{v}\left(y-\lambda\frac{\int_{\tau}^{y}xdF(x)}{F(y)-F(\tau)}\right)dF^{n-1}(y)\\<\;&v-\lambda\frac{\int_{\tau}^{v}xdF(x)}{F(v)-F(\tau)}.\end{aligned}$$

A.3 Proof of Corollary 1

After some algebric manipulations the bidding strategy given by equation (4) can also be written

$$\begin{aligned}\beta(v) =&(1+\lambda)v - \frac{1}{F^{n-1}(v)}\left(F^{n-1}(\tau)\tau + \int_\tau^v F^{n-1}(y)dy\right)\\ &- \frac{\lambda}{F^{n-1}(v)}\left(F^{n-1}(\tau)\tau + \int_\tau^v \mathbb{E}(X \mid \tau \leq X \leq y)dF^{n-1}(y)\right)\end{aligned}$$

for all $v \in [\tau, \overline{v}]$ and $\tau \in [\underline{v}, \overline{v}]$. Let us define for $\tau \in [\underline{v}, \overline{v}]$,

$$h(v) := \left(F^{n-1}(\tau)\tau + \int_\tau^v F^{n-1}(y)dy\right) \text{ and } k(v) := \left(F^{n-1}(\tau)\tau + \int_\tau^v \mathbb{E}(X \mid \tau \leq X \leq y)dF^{n-1}(y)\right).$$

Therefore,

$$h(\underline{v}) = \int_{\underline{v}}^v F^{n-1}(y)dy, h'(\tau) = \tau(F^{n-1}(\tau))' > 0, k(\underline{v}) = \int_{\underline{v}}^v \mathbb{E}(X \mid X \leq y)dF^{n-1}(y) \text{ and}$$

$$k'(\tau) = F^{n-1}(\tau) + \int_\tau^v \frac{f(\tau)}{F(y) - F(\tau)} \left(-\tau + \mathbb{E}(X \mid \tau \leq X \leq y)\right) dF^{n-1}(y) > 0.$$

Hence the bidding strategy is decreasing with the cut off τ for all $\tau \in [\underline{v}, \overline{v}]$, and the result follow.

A.4 Proof of Proposition 2

Part i) Following Bulow and Roberts (1989), for any auction, the *ex ante* expected surplus can be decomposed as the *ex ante* bidder's expected payoff and the expected payment, respectively denoted $EU(\tau)$ and $EP(\tau)$.

With signaling concerns, the *ex ante* expected surplus consists of the usual *ex ante* expected prize, $\int_\tau^{\overline{v}} vF^{n-1}(v)dF(v)$ and the *ex ante* expectation of the receiver's inference of the bidder's type. Because the receiver's Bayesian beliefs are a martingale, the *ex ante* expectation of the receiver's beliefs is independent of τ. The receiver's equilibrium beliefs are consistent with the observed auction's outcome, and the bidders take the expectation of these beliefs over the other bidder's possible types conditional on a bidder's own type, when determining the optimal bidding strategy. In this case, taking the *ex ante* expectation of these expectations with respect to the bidder's own type must necessarily result in the prior expectation $\mathbb{E}(V)$ by the law of total expectation, irrespective of the value of of φ, or equivalently τ. Therefore, the *ex ante* expectation of the receiver's equilibrium is independent of φ. Let us then write:

$$EP(\tau) = \int_\tau^{\overline{v}} vF^{n-1}(v)dF(v) + \lambda\mathbb{E}(V) - EU(\tau).$$

Note that the expected prize is decreasing with τ, and hence with φ. Using Lemma 2 which establishes $EU(\tau)$ increases with φ for λ sufficiently close to 1, it follows the expected payment decreases with φ for λ sufficiently close to 1. Therefore, the auction's expected revenue must be maximal at $\tau = \underline{v}$.

Part ii) Maximizing the auction's expected revenue means choosing $\tau = \underline{v}$. Of all φ that implement $\tau = \underline{v}$, expected revenue maximization requires choosing the φ at which the $\underline{v}$ type bidders are indifferent between participation and non-participation, i.e., the maximal entry fee that guarantees full participation, as defined in equation (3). An inspection of equation (3) shows that $\hat{\varphi}$ increases with λ.

A.5 Proof of Lemma 2

Using (4) and (5), we write the expected payoff π of a type v bidder, who pays φ, as:

$$\begin{aligned}
\pi(v) &\equiv F^{n-1}(v)\left(v(1+\lambda) - \beta(v)\right) + \lambda \int_v^{\overline{v}} \frac{\int_\tau^y x dF(x)}{F(y)-F(\tau)} dF^{n-1}(y) - \varphi \\
&= F^{n-1}(v)\left(v + \lambda \frac{F^{n-1}(\tau)}{F^{n-1}(v)}\tau - \frac{1}{F^{n-1}(v)} \int_\tau^v \left(y - \lambda \frac{\int_\tau^y x dF(x)}{F(y)-F(\tau)}\right) dF^{n-1}(y)\right) \\
&\quad + \lambda \int_v^{\overline{v}} \frac{\int_\tau^y x dF(x)}{F(y)-F(\tau)} dF^{n-1}(y) - F^{n-1}(\tau)\tau(1+\lambda) - \lambda \int_\tau^{\overline{v}} \frac{\int_\tau^y x dF(x)}{F(y)-F(\tau)} dF^{n-1}(y) \\
&\quad + \lambda \frac{\int_{\underline{v}}^\tau y dF(y)}{F(\tau)} \\
&= vF^{n-1}(v) - F^{n-1}(\tau)\tau - \int_\tau^v y dF^{n-1}(y) + \lambda \int_\tau^v \frac{\int_\tau^y x dF(x)}{F(y)-F(\tau)} dF^{n-1}(y) \\
&\quad + \lambda \int_v^{\overline{v}} \frac{\int_\tau^y x dF(x)}{F(y)-F(\tau)} dF^{n-1}(y) - \lambda \int_\tau^{\overline{v}} \frac{\int_\tau^y x dF(x)}{F(y)-F(\tau)} dF^{n-1}(y) + \lambda \frac{\int_{\underline{v}}^\tau y dF(y)}{F(\tau)} \\
&= vF^{n-1}(v) - F^{n-1}(\tau)\tau - \int_\tau^v y dF^{n-1}(y) + \lambda \frac{\int_{\underline{v}}^\tau y dF(y)}{F(\tau)} \\
&= \int_\tau^v F^{n-1}(y) dy + \lambda \frac{\int_{\underline{v}}^\tau y dF(y)}{F(\tau)}
\end{aligned}$$

The *ex ante* average expected payoff of a bidder, denoted $EU(\tau)$ is then

$$\begin{aligned}
EU(\tau) &= \lambda \frac{\int_{\underline{v}}^\tau y dF(y)}{F(\tau)} \int_{\underline{v}}^\tau dF(v) + \int_\tau^{\overline{v}} \pi(v) dF(v) \\
&= \lambda \frac{\int_{\underline{v}}^\tau y dF(y)}{F(\tau)} + \int_\tau^{\overline{v}} \int_\tau^v F^{n-1}(y) dy dF(v) \\
&= \lambda \frac{\int_{\underline{v}}^\tau y dF(y)}{F(\tau)} + \int_\tau^{\overline{v}} \left(\int_\tau^{\overline{v}} \mathbb{1}_{y \le v} f(v) dv\right) F^{n-1}(y) dy \\
&= \lambda \frac{\int_{\underline{v}}^\tau y dF(y)}{F(\tau)} + \int_\tau^{\overline{v}} (1-F(y)) F^{n-1}(y) dy
\end{aligned}$$

The derivative w.r.t. τ then becomes

$$\begin{aligned}\frac{dEU}{d\tau}(\tau) &= \lambda\left(\frac{\tau f(\tau)}{F(\tau)} - \frac{f(\tau)}{F(\tau)^2}\int_{\underline{v}}^{\tau} ydF(y)\right) - F^{n-1}(\tau)(1-F(\tau)) \\ &= \lambda\frac{f(\tau)}{F(\tau)}\left(\tau - \frac{\int_{\underline{v}}^{\tau} ydF(y)}{F(\tau)}\right) - F^{n-1}(\tau)(1-F(\tau)). \qquad (11)\end{aligned}$$

Equation (11) is positive for λ sufficiently close to 1 and higher for each larger level of λ.

A.6 Proof of Proposition 3

We proceed in 3 steps: 1) deriving the bidding function in Proposition 3, 2) showing the equilibrium bid β^{Id} is a global expected utility maximizing choice for each bidder, and 3) determining the equilibrium entry strategies.

Step 1: Using the same technical arguments than in the proof of Proposition 1 it follows that at the equilibrium $\beta^{Id}(\tau) = 0$. Computing the first-order condition from (6) and after imposing $\tilde{v} = v$ we obtain

$$(F^{n-1}(v)\beta^{Id}(v))' = v(F^{n-1}(v))' + (F^{n-1}(v))'\lambda\left(\int_{\tau}^{\bar{v}} \mathbb{E}(V|V \geq y)dF^{n-1}(y) - \int_{\tau}^{\bar{v}} \mathbb{E}(V|V \leq y)dF^{n-1}(y)\right).$$

After integrating and using $\beta^{Id}(\tau) = 0$, the bidding strategy at equilibrium follows.

Step 2: To show the bidding strategy is optimal for all bidders we use the same technical arguments than in the Chapter 2 of Krishna (2009). Remark that the bidding strategy is a strictly increasing function. Suppose all bidders follows the bidding strategy β^{Id}, and that a bidder with a type v deviates from the equilibrium by submitting $\beta^{Id}(z)$ instead of $\beta^{Id}(v)$. Let us denote $\mathbb{E}U(\beta^{Id}(z), v)$ the expected payoff of type v bidder with a such bidding behavior. It follows that

$$\begin{aligned}\mathbb{E}U(\beta^{Id}(z), v) &= F^{n-1}(z)(v - \beta^{Id}(z)) + F^{n-1}(z)\lambda\int_{\tau}^{\bar{v}} \mathbb{E}(V|V \geq y)dF^{n-1}(y) \\ &\quad -(1 - F^{n-1}(z))\lambda\int_{\tau}^{\bar{v}} \mathbb{E}(V|V \leq y)dF^{n-1}(y) - \varphi^{Id} \\ &= F^{n-1}(z)(v - z) - F^{n-1}(\tau)\tau + \int_{\tau}^{z} F^{n-1}(x)dx + \lambda\int_{\tau}^{\bar{v}} \mathbb{E}(V|V \leq y)dF^{n-1}(y) - \varphi^{Id}\end{aligned}$$

Using the bidding strategy at the equilibrium and integrating by part, we obtain the second line. Therefore,

$$\mathbb{E}U(\beta^{Id}(v),v)-\mathbb{E}U(\beta^{Id}(z),v) = F^{n-1}(z)(z-v)-\int_{\tau}^{z} F^{n-1}(x)dx \geq 0 \text{ for all } z \text{ such as } v \leq z \text{ or } v \geq z.$$

Step 3: Using similar arguments than for the case with winner's identity and its payment revealed to determine the equilibrium entry strategies.

If the cut off type τ stays out, he pools with all the non-participating lower bidders and he is perceived by the receiver as $\lambda\frac{\int_{\underline{v}}^{\tau} vdF(v)}{F(\tau)}$. If the τ type decides to pay the entry fee, he wins the auction with probability $F^{n-1}(\tau)$ with a zero bid and he gets the object he values τ. However, as his payment cannot be identified, he obtains an expected payoff from the receiver's inferences equal to $\lambda\int_{\tau}^{\bar{v}}\mathbb{E}(V|V\geq y)dF^{n-1}(y)$. Otherwise, he gets the expected inference of a losing participating bidder $\int_{\tau}^{\bar{v}}\mathbb{E}(V|V\leq y)dF^{n-1}(y)$. Therefore, the equilibrium entry strategies for an internal $\tau\in(\underline{v},\overline{v})$ are characterized by the relationship between the entry fee φ^{Id} and the cut off type τ described by the equation (7).

A.7 Proof of Corollary 2

The derivative of the bidding strategy w.r.t. τ equals to

$$(F^{n-1}(\tau))'\left(-\frac{\tau}{F^{n-1}(v)}+\lambda\mathbb{E}(V|V\leq\tau)-\lambda\mathbb{E}(V|V\geq\tau)\right)<0 \text{ for all } \tau\in[\underline{v},\overline{v}].$$

A.8 Proof of Proposition 4

We now proceed in 3 steps: 1) deriving the bidding function in Proposition 4, 2) showing the equilibrium bid β^A is a global expected utility maximizing choice for each bidder, and 3) determining the equilibrium entry strategies.

Step 1: Using the same technical arguments than in the proof of Proposition 1 it follows that at the equilibrium $\beta^A(\tau)=0$. Computing the first-order condition from (8) and after imposing $\tilde{v}=v$ we obtain

$$(F^{n-1}(v)\beta^A(v))' = v(F^{n-1}(v))'+\lambda.$$

After integrating and using $\beta^A(\tau)=0$, the bidding strategy at equilibrium follows.

Step 2: To show the bidding strategy is optimal for all bidders we use the same technical arguments than in the Chapter 2 of Krishna (2009). Suppose all bidders follows the bidding strategy β^{Id}, and that a bidder with a type v deviates from the equilibrium by submitting $\beta^A(z)$ instead of $\beta^A(v)$. Let us denote $\mathbb{E}U(\beta^A(z),v)$ the expected payoff of type v bidder with a such bidding behavior. It follows that

$$\begin{aligned}\mathbb{E}U(\beta^A(z), v) &= F^{n-1}(z)(v - \beta^A(z)) + \lambda z - \varphi^A \\ &= F^{n-1}(z)(v - z) - F^{n-1}(\tau)\tau + \int_\tau^z F^{n-1}(x)dx + \lambda\tau - \varphi^A\end{aligned}$$

Using the bidding strategy at the equilibrium and integrating by part, we obtain the second line. Therefore,

$$\mathbb{E}U(\beta^A(v), v) - \mathbb{E}U(\beta^A(z), v) = F^{n-1}(z)(z - v) - \int_\tau^z F^{n-1}(x)dx \geq 0 \text{ for all } z \text{ such as } v \leq z \text{ or } v \geq z,$$

which establishes that if all bidders except one follow the bidding strategy β^A, then it is optimal for a type v bidder to bid $\beta^A(v)$.

However this is true if $(\beta^A(.))' > 0$. The derivative of the biding strategy is given by

$$(\beta^A(v))' = \frac{d}{dv}\left(\frac{1}{F^{n-1}(v)}\int_\tau^v y dF^{n-1}(y)\right) + \frac{\lambda}{F^{n-1}(v)} - \frac{(v-\tau)(F^{n-1}(v))'}{(F^{n-1}(v))^2},$$

and is strictly positive if F^{n-1} is concave.

Step 3: We are using similar arguments than in the two previous information disclosure policies to determine the equilibrium entry strategies.

If the cut off type τ stays out, he pools with all the non-participating lower bidders and he is perceived by the receiver as $\lambda\frac{\int_{\underline{v}}^{\tau} v dF(v)}{F(\tau)}$. If the τ type decides to pay the entry fee, he wins the auction with probability $F^{n-1}(\tau)$ with a zero bid, in which case he gets the object he values τ and he is inferred as type τ. Otherwise he is a participating loser with the probability $1 - F^{n-1}(\tau)$ and is also perceived as type τ by the receiver. Hence, the equilibrium entry strategies for an internal $\tau \in (\underline{v}, \overline{v})$ are characterized by the relationship between the entry fee φ^A and the cut off type τ described by the equation (9).

A.9 Derivations for the second-price auction and the English auction

In this appendix, let us assume for simplicity that F is the uniform distribution over the unit interval and that $\lambda = 1$.

Second-price auction

For the second-price auction, Following Bos and Truyts (2021) and assuming the existence of a strictly increasing equilibrium bidding function, the problem of a type v

bidder choosing which type $\tilde{v}$ to imitate reads:

$$\tilde{v}^{n-1}v - \int_{\tau}^{\tilde{v}} \beta(y)dy^{n-1} + \int_{\tau}^{\tilde{v}} \frac{\int_x^1 ydy}{1-x}dx^{n-1} + \tau^{n-1}\frac{\int_\tau^1 xdx}{1-\tau} - \varphi$$
$$+ (n-1)(1-\tilde{v})\sum_{i=0}^{n-2} \begin{pmatrix} n-2 \\ i \end{pmatrix} \frac{\tau^{n-2-i}(\tilde{v}-\tau)^i}{i+1}\left(\tilde{v} + i\frac{\int_\tau^{\tilde{v}} ydy}{\tilde{v}-\tau}\right)$$
$$+ \int_{\tilde{v}}^1 \left(\frac{\sum_{i=1}^{n-2} \begin{pmatrix} n-2 \\ i \end{pmatrix} \frac{\tau^{n-2-i}(y-\tau)^i}{i+1}\left(y + i\frac{\int_\tau^y xdx}{y-\tau}\right)}{y^{n-2}-\tau^{n-2}} \right) d\left((n-1)y^{n-2} - (n-2)y^{n-1}\right).$$

$$= \tilde{v}^{n-1}v - \int_{\tau}^{\tilde{v}} \beta(y)\,dy^{n-1} + \frac{\tilde{v}^{n-1}-(\tau)^{n-1}}{2} + \frac{n-1}{2n}\left(\tilde{v}^n - (\tau)^n\right) + (\tau)^{n-1}\frac{1+\tau}{2} - \varphi$$
$$+ (n-1)(1-\tilde{v})\left(\sum_{i=0}^{n-2}\begin{pmatrix} n-2 \\ i \end{pmatrix}(\tau)^{n-2-i}(\tilde{v}-\tau)^i\left(\frac{\tilde{v}+i\frac{\tilde{v}+\tau}{2}}{i+1}\right)\right)$$
$$+ \int_{\tilde{v}}^1 \frac{(n-1)(n-2)\left(y^{n-3}-y^{n-2}\right)}{y^{n-2}-(\tau)^{n-2}} \sum_{i=1}^{n-2}\begin{pmatrix} n-2 \\ i \end{pmatrix}(\tau)^{n-2-i}(y-\tau)^i\left(\frac{y+i\frac{y+\tau}{2}}{i+1}\right)dy.$$

Note then that

$$\begin{aligned} & \sum_{i=0}^{n-2}\begin{pmatrix} n-2 \\ i \end{pmatrix}(\tau)^{n-2-i}(\tilde{v}-\tau)^i\frac{1}{i+1} \\ = & \frac{1}{(n-1)(\tilde{v}-\tau)}\sum_{i=0}^{n-2}\frac{(n-1)!}{(i+1)!\,(n-1-(i+1))}(\tau)^{n-1-(i+1)}(\tilde{v}-\tau)^{i+1} \\ = & \frac{\tilde{v}^{n-1}-(\tau)^{n-1}}{(n-1)(\tilde{v}-\tau)}. \end{aligned}$$

Since $\frac{i}{1+i} = 1 - \frac{1}{1+i}$ we have $\sum_{i=0}^{n-2}\begin{pmatrix} n-2 \\ i \end{pmatrix}(\tau)^{n-2-i}(\tilde{v}-\tau)^i\frac{i}{i+1} = \tilde{v}^{n-2} - \frac{\tilde{v}^{n-1}-(\tau)^{n-1}}{(n-1)(\tilde{v}-\tau)}$.

Using similar algebraic manipulations for $\sum_{i=1}^{n-2}\begin{pmatrix} n-2 \\ i \end{pmatrix}\tau^{n-2-i}(y-\tau)^i$, the problem can be written after some computations

$$
\begin{aligned}
= \quad & \tilde{v}^{n-1}v - \int_{\tau}^{\tilde{v}} \beta(y)\, dy^{n-1} + \frac{n+1}{2}\tilde{v}^{n-1} - \frac{n-1}{2}\tau\tilde{v}^{n-1} - \frac{n^2-n+1}{2n}\tilde{v}^{n} \\
& + \frac{\tilde{v}(\tau)^{n-1}}{2} + \frac{(n-1)}{2}\tilde{v}^{n-2}\tau \\
& - \frac{n-1}{2n}(\tau)^{n} + (\tau)^{n-1}\frac{1+\tau}{2} - (\tau)^{n-1} - \varphi \\
& + \int_{\tilde{v}}^{1} \frac{(n-1)(n-2)\left(y^{n-3}-y^{n-2}\right)}{y^{n-2}-(\tau)^{n-2}} \left(\frac{y^{n-1}-\tau^{n-1}}{2(n-1)} - (\tau)^{n-2}y + y^{n-2}\frac{y+\tau}{2} \right) dy
\end{aligned}
$$

Deriving the first order condition for $\tilde{v}$, imposing $\tilde{v} = v$ and after a series of algebraic manipulations, we obtain the following bidding function

$$
\begin{aligned}
\beta(v) = v + \frac{1}{2} - \frac{1}{2}\left(\frac{v}{(n-1)} - \frac{\tau^{n-1}}{v^{n-2}(n-1)} + \tau \right) + \frac{(1-v)}{2}\left(n + \frac{(n-2)\tau}{v} \right) \\
- \frac{(n-2)(1-v)}{\left(v^{n-2}-\tau^{n-2}\right)v}\left(\frac{v^{n-1}-\tau^{n-1}}{2(n-1)} - \tau^{n-2}v + \frac{v+\tau}{2}v^{n-2} \right).
\end{aligned}
$$

Note that if $n = 3$, $\beta(v)$ reduces to

$$
\beta(v) = \frac{5-3\tau}{4} + \frac{(\tau+1)\tau}{4v},
$$

which decreases with v, and thus contradicts the initial assumption of a strictly increasing bidding function, such that a fully separating PBE does not exist for $n = 3$. In general, this bidding function tends to be undefined at τ, and this impedes the characterization of the optimal entry fee for the second-price auction.

English auction

We consider an English auction as in Bos and Truyts (2021), i.e., a *minimal button auction* but with an entry fee. Bidders who have paid the fee know at each price only whether or not there are still two or more bidders active or not. Bidders do not know the number of others who have effectively paid the fee, but the receiver does observe whether a bidder has participated or not.

Given that a bidder has paid the fee, he compares at each price b his expected payoffs if winning and if losing the auction at this price. Assuming a strictly increasing exit rule β, a type v bidder chooses to exit at the price where the expected payoffs of winning and losing are equal:

$$
\begin{aligned}
& v-b+\frac{\int_{\beta^{-1}(b)}^{1} x dx}{1-\beta^{-1}(b)} \\
= & \frac{1}{\left(\beta^{-1}(b)-\tau\right)^{n-2}} \sum_{i=0}^{n-2}\binom{n-2}{i}(\tau)^{n-2-i}\left(\beta^{-1}(b)-\tau\right)^{i}\left(\frac{\beta^{-1}(b)+i \frac{\int_{\tau}^{\beta^{-1}(b)} x dx}{\beta^{-1}(b)-\tau}}{i+1}\right),
\end{aligned}
$$

such that the optimal exit strategy can be solved as:

$$
\begin{aligned}
\beta(v) & = v+\frac{1+v}{2}-\frac{1}{(v-\tau)^{n-2}}\left(\left(\frac{v^{n-1}-(\tau)^{n-1}}{(n-1)(v-\tau)}\right) v+\left(v^{n-2}-\frac{v^{n-1}-(\tau)^{n-1}}{(n-1)(v-\tau)}\right) \frac{v+\tau}{2}\right) \\
& = \frac{1+3 v}{2}-\frac{1}{(v-\tau)^{n-2}}\left(\frac{v^{n-1}-\tau^{n-1}}{2(n-1)}+\frac{v^{n-1}+\tau v^{n-2}}{2}\right).
\end{aligned}
$$

However, this exit rule can be shown to be non-increasing for a nontrivial subset of the type space, which contradicts the above assumption of a strictly increasing β, and demonstrates the non-existence of a fully separating equilibrium. For instance, for $n=3$, $\beta(v)$ reduces to

$$
\beta(v)=\frac{\left(3 v^{2}-8 v \tau+2 v+\tau^{2}-2 \tau\right)}{4(v-\tau)},
$$

which is non-monotonic with respect to v on $[\tau, 1]$. For $n=4, \beta(v)$ becomes

$$
\beta(v)=\frac{1+3 v}{2}-\frac{1}{(v-\tau)^{2}}\left(\frac{v^{3}-\tau^{3}}{6}+\frac{v^{3}+\tau v^{2}}{2}\right),
$$

which is equally non-monotonic w.r.t. v on $[\tau, 1]$.

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