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Booster Draft Mechanism for Multi-Object Assignment

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Booster Draft Mechanism for Multi-Object Assignment*

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Abstract

We describe a new mechanism - what we call a booster draft - for allocating multiple, indivisible objects among a group of individuals. The mechanism's appeal lies in its strategy-proofness and simplicity: Individuals take turns drawing objects from different sets - called boosters - and simply need to identify their favorite object when it's their turn to choose. Following a market design approach, we examine how to tailor the booster draft mechanism to specific multi-object assignment problems. As an illustrative example, we consider the assignment of teaching positions to graduate students. We show that, through the right design of the boosters, not only is the mechanism strategy-proof, but the resulting allocations are fair and efficient. In fact, in the described domain, under some additional mild axioms, any strategy-proof mechanism is some variation of a booster draft. Finally, using data on graduate students preferences, we demonstrate that the booster draft is useful and easy to implement in practice.

Keywords: Matching, Envy-free, Booster Draft, Multi-Object Assignment

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1 Introduction

In this paper, we study the multi-object assignment problem. That is, $m \times n$ indivisible objects have to be distributed among n individuals, without the use of monetary transfers. Examples include the assignment of shifts to interchangeable workers, players to sport teams, courses to students, and teaching assignments to graduate students. We introduce a new mechanism, which stands out in terms of simplicity and strategy-proofness, that is, it will be always optimal for individuals to reveal their true preferences. The mechanism is inspired by the following multi-object assignment process used in Magic: The Gathering, a competitive trading card game:[1]

> `In a booster draft, players each receive three booster packs of 15 cards. After being seated around a table, each player simultaneously opens one booster pack, selects a single card, and then passes the rest over to the next player. After all players have drafted fifteen cards, they each open their second pack, and drafting continues, sometimes in reverse order during the second pack (MTGWiki, 2019).`

We formalize an algorithm that captures the essential elements of the mechanism described above. As input, individuals report rankings over the available objects. In a first step, the objects are arbitrarily divided into m sets of size n, and a separate priority order for each set determines the sequence in which objects are picked. In line with the motivation, we refer to these sets as "boosters." Following his/her reported ranking, the individual whose turn it is to select from a given booster adds the best available object to his/her collection. Once all objects are distributed, a final allocation is reached. Fixing the objects within each booster as well ass the corresponding priority orders the algorithm induces a function from rankings to allocations, which we refer to as a booster draft (BD) mechanism. More precisely, we introduce a class of mechanisms, as every design of boosters and priorities yields a distinct BD mechanism.

We start by analyzing how well BD mechanism perform if used for a general multi-object assignment problem. Postponing the details to a later part of the introduction, we show that any BD mechanism is strategy-proof in the "responsive" preference domain.[2] To ensure that individuals cannot benefit from manipulation, we need them to pick at most once from every booster, therefore we restrict the sets to be of size n. Moreover, responsiveness ensures that an individual does not want to change its earlier picked objects, based on the objects received later on. Furthermore, we show that any "balanced" booster drafts satisfies a reasonable notion of fairness.[3] It is well known that in the multi-object environment,

[1]Magic: The Gathering (MTG) was the first commercially successful trading card game, developed by Richard Garfield and published in 1993 by Wizards of the Coast (https://company.wizards.com/). Analogous to a sports drafts, MTG introduced a play mode in which players draft cards from a common pool. Afterwards every player builds a deck using a subset of the drafted cards. Only then do players compete against each other with their constructed decks. There are countless other online/offline trading card games featuring a booster draft-inspired play-mode.

[2]Responsiveness requires that an individual will always prefer a higher ranked object to a lower ranked one regardless of any other items in his/her possession.

[3]Balanced BD mechanisms place the following restriction on the priority orders: For any pair of individuals i and j, i

there does not exist any mechanism simultaneously satisfying strategy-proofness, efficiency, and fairness. Therefore, we cannot ensure that the outcomes of the BD mechanisms will be efficient.

Following these observations, in the second part of the paper, we take a market design perspective, exploiting additional restrictions on preferences and allocations for specific markets, in order to improve the outcome of the BD mechanism. Paying attention to detail, we ask how to design the underlying boosters and priority orders. We focus our attention on the assignment of teaching positions to graduate students, in which a restriction is placed on the set of feasible allocations. Specifically, at the economics department under consideration, graduate students are supposed to work as a teaching assistant for exactly one fall and one spring semester course.[4] Here, the optimal way of creating boosters is to group up all spring teaching assignments in one booster and all fall semester assignments in the other. In this case, the balanced booster draft is not only strategy-proof, but also (weakly) efficient and fair. More generally, we describe the *partition-restricted* assignment domain, i.e., any multi-object assignment problem for which there exists an exogenous partition of objects, such that any two objects within the same set cannot be obtained by the same individual. As before, the creation of boosters for running the BD algorithm is no longer arbitrary, but naturally follows the exogenously given partition. In theorem 1, we characterize the set of all "partition-consistent" BD mechanism for this domain. In particular, given the standard requirements of non-bossiness and neutrality any strategy-proof mechanism must fall into the category of booster draft mechanisms.[5] We conclude that in any partition-restricted multi-object assignment problem, the balanced BD mechanism arises as a natural candidate to be employed. How to create boosters for other multi-object assignment problems remains an open question.

In the third part of the paper, we take a closer look at the assigning of graduate students to teaching positions. We use 2018 data on the preferences of graduate students, at a particular economics department, to simulate assignments under the balanced BD mechanism. That is, at the end of the academic year, students separately rank all the fall and spring semester tasks. Before the start of the new academic year, an assignment is created based on the submitted rankings. For the simulation, students are randomly ordered and pick their preferred fall semester assignments one after another. Then the initial random order is reversed and students pick their spring semester assignments in the same fashion. The BD mechanism outperforms the actual assignment in that year, both in terms of efficiency and fairness. Moreover, simulating the outcome of serial dictatorship as an alternative, we show that the BD mechanism reaches comparable outcomes in terms of efficiency, while achieving higher

has lower priority than j in at most half of the available boosters, rounded down if there is an odd number of boosters, i.e., $\lceil \frac{m}{2} \rceil$.

[4]One relevant rational for the requirement is the following: As many graduate students in the economics department are international students, their visa status (F1) allows them to work up to 20 hours per week, preventing them from fulfilling the work-requirement for two positions in the same semester, without violating their visa regulations.

[5]Non-Bossiness requires that no individual can influence the allocation of another individual without affecting its own allocation, while neutrality states that the mechanism should be immune to a relabeling of the object.

fairness.[6] This discussion concludes the main part of the paper. We belief that an additional strength of the booster draft lies in its simplicity. Therefore, in the remainder, we evaluate in which sense the BD rule's non-manipulability is simple to grasp, following the concept of obvious strategy-proofness.[7] We now will discuss some of the previously omitted definitions and ideas in more detail.

Running the BD mechanism, every individual reports a simple order/ranking over the available objects.[8] As strategy-proofness, efficiency, and fairness are all formulated in terms of individuals preferences, we first need to establish a link between the reported order over objects and the underlying preferences. In particular, consider the following partial order, which we refer to as a dominance relation: Given a simple order over objects, for any two (same size) sets A and B, A dominates B if for every object in B there is a concomitant object in A that is (weakly) preferred to the one in B. In lemma 1, we show that if A is preferred to B by set-wise domination, then the same holds true for the actual preferences. This connection allows us to analyze which properties the BD mechanism satisfies.

A mechanism is strategy-poof if an individual cannot obtain a better outcome by submitting a untruthful ranking over objects. The BD mechanism is strategy-proof, as the final allocation is (weakly) set-wise dominating any other outcome obtainable by submitting an different ranking. The idea behind the fairness concept is as follows: Pick a final allocation and suppose individual i prefers individual j's bundle to his/hers. Let us sequentially remove the best object from j's and the worst object from i's assignment, following i's simple order. At some point, i (weakly) prefers her reduced bundle to j's reduced bundle. A mechanism is k *envy-free*, if the maximum number of pairs of objects that have to be removed to eliminate envy for any individual i over the bundle of any j, at any possible allocation, is equal to k. The larger k is, the higher the envy of an individual. The maximum envy under the balanced BD mechanism is equal to half of the obtained objects rounded up.

Efficiency requires that for any final allocation, no other allocation of objects makes everyone weakly and at least one individual strictly better off. We relax efficiency to dominance efficiency, ruling out that all individuals can be made better off in terms of set-wise dominance. In proposition 4 we show that an allocation is dominance efficient if and only if there does not exist any "exchange-cycles" between a subset of individuals, s.t. everyone gives and receives exactly a single object, and everyone is better off after the trade takes place. However, our weakened efficiency criteria does not rule out that swapping a combination of objects, some deemed better and some worse than the ones exchanged, can lead to a more desired allocation for all involved parties. The BD mechanism violates dominance efficiency in

[6] Under a serial dictatorship, for any two students A and B, one of them is going to choose all his/her objects before the other. The mechanism is simple to implement, efficient, and strategy-proof and has therefore often been used in practice. An important shortcoming is that serial dicatorships lead to very unfair allocations, especially if individuals value similar objects.

[7] In practice strategy-proofness is not always strong enough. People sometimes will try to manipulate a mechanism, failing to recognize its strategy-proofness. Li (2017) introduces the strengthening of strategy-proofness called obvious strategy-proofness that addresses the issue.

[8] For any preference relation, we refer to the ranking of singleton sets as the "underlying simple order."

the standard responsive domain. We also show that in the standard responsive preference domain no mechanism can simultaneously satisfy envy-freeness for half of the objects, dominance strategy-proofness, and dominance efficiency.[9] As discussed previously, we can avoid the impossibility result by incorporating market specific restrictions. For instance, we show that in the partition-restricted assignment domain the BD mechanism satisfies dominance efficiency on top of strategy-proofness and envy freeness for half of the objects.

Finally, we ask whether the BD mechanism is implementable via an extensive form game in an obviously strategy-proof (OSP) way. A mechanism is OSP implementable if there exists an extensive form game that yields the same outcome as the proposed mechanism with the added restriction that, at any information set in which an individual is called to play, the best outcome under truthful play is weakly preferred to any possible history reachable from the same information set. The BD mechanism is not OSP implementable. We introduce a weakening of OSP called *dominance obvious strategy-proofness* (DOSP), that limits the attention to outcome pairs comparable by set-wise domination. Unlike the standard responsive domain, the BD mechanism is DOSP implementable in the partition-restricted domain, providing additional evidence that the BD mechanism is a strongly viable candidate for the restricted multi-object assignment problems. We continue by discussing the relevant literature.

2 Related Literature

A series of impossibility results have pointed out that any efficient and strategy-proof mechanism is a serial dictatorship (Pápai, 2001; Klaus and Miyagawa, 2002; Ehlers and Klaus, 2003). We add a new impossiblity result to the literature, showing that even weaker versions of fairness, strategy-proofness, and efficiency cannot be simultaneously satisfied by a mechanism.

Initially, we are interested in strategy-proof mechanisms that are also fair. Related to this, Moulin (2019) provides a comprehensive survey of the long-standing literature on fair division problems. Our definition of envy-freeness is adapted from Budish (2011), although we modify their definition to account for the possibility of distributing bads, as well extending it, allowing for the removal of an arbitrary number of objects.[10] For practicality, we let individuals report simple orders over objects. We then have to establish a link between the reports of individuals and their preferences across sets of objects. This approach relates to Brams and Fishburn (2000), Brams et al. (2003), and Edelman and Fishburn (2001).

[9]Dominance strategy-poofness is a weakening of strategy-proofness, requiring that no individual can manipulate the mechanism, s.t. his/her assignment (strictly) improves under the dominance relation. Hence, this notion allows for some manipulations to take place.

[10]Suppose for example that only non-disposable bads are distributed. Hence removing a bad from the bundle of individual j will only increase the envy of individual i. In this case one should remove the worst object from i's bundle instead. We take care of both cases by always removing the best object from j and the worst object from i simultaneously. Aziz et al. (2018) have a similar definition based on removing a single good on each side. Moreover, instead of considering 1 envy-freeness, we allow for an arbitrary k.

In the second part of the paper, we take a market design perspective. We exploit the additional structure specific markets impose on preferences and or final allocations, to adjust our mechanism to the problem at hand. This has been done before for multi-unit assignment problems, in the context of course allocation at business schools (Sönmez and Ünver, 2010; Budish, 2011; Budish and Cantillon, 2012).[11] Budish (2011) provides a solution to the more general combinatorial assignment problem, introducing the approximate competitive equilibrium from equal incomes (ACEEI) mechanism. Efficiency and strategy-proofness of the ACEEI mechanism rely on the market being large enough such that people become price takers. Unfortunately, ACEEI cannot be obtained in a constructive way (Othman et al., 2010; Budish et al., 2016). This might cause legitimacy issues (Bo and Li, 2019), as its not possible to publicly implement the outcome of ACEEI. Related to this, Li (2017) points out that if a mechanism is hard to understand in practice, some individuals will employ dominated strategies, even if the mechanism is strategy-proof. An additional appeal of the BD mechanism lies in its simplicity. The discussion of the dominance obviously strategy-proofness, relates to the small body of literature on obviously strategy-proofness (Li, 2017; Zhang and Levin, 2017; Ashlagi and Gonczarowski, 2018; Pycia and Troyan, 2018; Troyan, 2016).

In general, our research relates to the larger field of matching theory started by Gale and Shapley (1962). In particular, the characterization result draws from Svensson (1999) and Hatfield (2009), while the responsiveness preference assumption is based on Roth (1985). Finally, the assignment of graduate students to teaching assignments falls into the category of applied matching problems (Abdulkadiroglu and Sönmez, 2003; Sönmez and Switzer, 2013; Delacrétaz et al., 2019). We are not aware that this particular application has been discussed in any previous literature.

3 Model

A **multi-object assignment problem** is a triple $\langle I, O, \succsim \rangle$, where

1. I is a finite set of $|I| = n$ individuals,
2. O is a finite set of $|O| = m \times n$ objects with $m \geq 2$, and
3. $\succsim = (\succsim_i)_{i \in I}$ a list of preferences over sets of objects 2^O.

We want to distribute all the available objects among the individuals. An **allocation** $A = (A_i)_{i \in I}$ gives every individual $i \in I$ a subset of objects $A_i \in 2^O$. An allocation is **feasible** if for any two distinct individuals $i, j \in I$ with $i \neq j$, their assignments do not overlap $A_i \cap A_j = \varnothing$, and all objects are distributed $\bigcup_{i \in I} A_i = O$. Let $\mathcal{A}$ denote the set of all feasible allocations. Every individual has a

[11] The multi-unit assignment problem is a slight variation to on the multi-object assignment problem, in which several units of the same object are available, e.g., representing the number of available seats for each course. None of the results we present depend on the absence of multiple copies of objects. We ignore it to reduce the notation, as it does not give any additional insight.

preference relation $\succsim_i$ over sets of objects.[12] Slightly abusing notation $\succsim_i$ denotes preferences over sets of objects as well as allocations, such that $A \succsim_i A'$ if and only if $A_i \succsim_i A'_i$. The underlying assumption is that individuals only care about their own assignment.

From individual i's perspective, an object o is a **good** if receiving the object is preferred to not receiving it, i.e., $\{o\} \succ_i \varnothing$. The empty-set, $\varnothing$, represents an empty assignment. Conversely, an object is a **bad** if not receiving it is preferred to receiving it, i.e., $\varnothing \succ_i \{o\}$. For simplicity, we focus on the case where everyone agrees whether objects are good or bad. Formally, we have $\{o\} \succsim_i \varnothing$ if and only if $\{o\} \succsim_j \varnothing$ for all $j \in I$ and hence $\varnothing \succ_i \{o\}$ if and only if $\varnothing \succ_j \{o\}$ for all $j \in I$. Preferences over objects are **responsive**. The idea is that if an individual prefers object o to another object o' then we can infer, no matter what other objects $O' \subset O$ the individual possesses, that $\{o\} \cup O'$ is preferred to $\{o'\} \cup O'$. I.e., for any $o, o' \in O$ and $O' \subset O \setminus \{o, o'\}$ we have that $O' \cup \{o\} \succsim_i O' \cup \{o'\}$ if and only if $\{o\} \succsim_i \{o'\}$. Likewise, a good is always desirable, while a bad makes an individual always worse off, i.e., for any $o \in O$ and $O' \subset O \setminus \{o\}$ we have $O' \cup \{o\} \succ_i O'$ if and only if $\{o'\} \succ_i \varnothing$. Finally, we restrict our attention to preferences that strictly rank any pair of singletons. That is for any $o, o' \in O$ with $o \neq o'$ either $\{o\} \succ_i \{o'\}$ or $\{o'\} \succ_i \{o\}$.

3.1 Submitted Rankings and the Booster Draft Mechanism

As it is impractical to ask individuals for their full preference relation over all sets of objects, throughout the analysis, we let each individual report a **strict simple order** P_i over the available objects O, with the associated simple order R_i.[13] The set of possible rankings for any $i \in I$ is denoted as $\mathcal{P}_i$, and represents all possible ways one can order the available objects O. $P = (P_i)_{i \in I}$ denotes a list of simple orders for every individual $i \in I$ with $\mathcal{P}$ representing the set of all possible lists. For a given preference relation $\succ_i$ over 2^O we say P_i is the **associated simple order** over O if for all $o, o' \in O$ we have that $o \; P_i \; o'$ if and only if $\{o\} \succ_i \{o'\}$. That is, the associate simple order ranks all the objects in the same way as the underlying preference relation.[14]

Even though it is not possible to infer the whole preference relation $\succsim_i$ over 2^O from the associated simple order P_i over O, the responsiveness assumption lets us compare some sets by element-wise dominance. To express this relationship, we define a partial order $\geq_i$ over 2^O based on a simple order P_i. We will refer to the partial order $\geq_i$ as the **dominance relation**. The idea is that two sets of objects

[12] For any $O', O'' \in 2^O$ with $O' \succsim O''$ but $O'' \not\succsim O'$ we write $O' \succ_i O''$, similarly for any $O', O'' \in 2^O$ with $O' \succsim O''$ but $O'' \succsim O'$ I write $O' \sim_i O''$.

[13] The strict simple order P_i is *transitive*, *asymmetric*, and *complete*. The associated simple order R_i is *transitive*, *antisymmetric* and *strongly complete*. Strong completeness implies reflexiveness and is therefore not listed under the properties of a simple order. Simply put, unlike P_i which is asymmetric, R_i also compares any object with itself. Otherwise both relations ranks every pair in the same way. See Roberts (1985) for an excellent overview on binary relations and their properties. Finally as every person agrees whether an object is a good or a bad, together with requiring every object to be assigned to someone, it is sufficient to let individuals rank O as opposed to $O \cup \{\varnothing\}$.

[14] Note that P_i is a strict simple order (*transitive*, *asymmetric*, and *complete*) as $\succ_i$ strictly ranks all pairs of singleton sets in a transitive way. Moreover, the associated simple order P_i is uniquely determined for each preference relation $\succ_i$, while multiple (responsive) preference relations $\succ_i$ are consistent with any given simple order P_i.

are comparable if for every object in one set we can find a weakly preferred concomitant object in the other set. Formally, let $o'_{i,l} = \{o \in O' : |\{o' \in O' : o' \; R_i \; o\}| = l\}$ be the lth best object in subset $O' \in 2^O$ following simple order P_i. Then, for any two subsets $O', O'' \in 2^O$ of equal size $|O'| = |O''| = m'$, we have $O' \geq_i O''$ if and only if $o'_{i,l} \; R_i \; o''_{i,l}$ for all $l \in \{1, \ldots, m'\}$.[15] In the following we show that if the dominance relation $\geq_i$ based on P_i rank two sets of objects, then it does so in the same way as the responsive preference relation with the same associated simple order P_i.

Lemma 1. *Let $\succsim_i$ be any responsive preference relation over 2^O with associated simple order P_i, and $\geq_i$ the corresponding dominance relation. For any $O', O'' \in 2^O$ if $O' \geq_i O''$ then $O' \succsim_i O''$.*

Proof. Suppose we have $O', O'' \in 2^O$ with $O' = \{o'_{i,1}, \ldots, o'_{i,m'}\} \geq_i O'' = \{o''_{i,1}, \ldots, o''_{i,m'}\}$. As $O' \geq_i O''$ we have $o'_{i,1} \; R_1 \; o''_{i,1}$ as well as $\{o'_{i,1}\} \succsim_i \{o''_{i,1}\}$. Using responsiveness for $\{o''_{i,2}, \ldots, o''_{i,m'}\} \subseteq O \setminus \{o'_{i,1}, o''_{i,1}\}$ and $\{o'_{i,1}\} \succsim_i \{o''_{i,1}\}$ we get $\{o'_{i,1}, o''_{i,2}, \ldots, o''_{i,m'}\} \succsim_i O'' = \{o''_{i,1}, o''_{i,2}, \ldots, o''_{i,m'}\}$. Replacing one-by-one $o''_{i,k}$ by $o'_{i,k}$ for all $k \in \{2, \ldots, m'\}$ and invoking responsiveness we get $O' = \{o'_{i,1}, o'_{i,2}, \ldots, o'_{i,m'}\} \succsim_i \ldots \succsim_i \{o'_{i,1}, o''_{i,2}, \ldots, o''_{i,m'}\} \succsim_i \{o''_{i,1}, o''_{i,2}, \ldots, o''_{i,m'}\} = O''$. By transitivity of $\succsim_i$ we reach the conclusion that $O' \succsim_i O''$. □

We now go back to the question of how to distribute the available objects. That is, we are interested in finding a **simple mechanism** $\psi : \mathcal{P} \to \mathcal{A}$ that selects an allocation $A \in \mathcal{A}$ for any reported list of orderings $P \in \mathcal{P}$.

Booster Draft (BD) Algorithm

Step 0.

Let the set of objects O be arbitrarily partitioned into m boosters of equal size $\{O^1, \ldots, O^m\}$ with $|O^k| = n$ for all $k \in \{1, \ldots, m\}$. Moreover construct m different priority for every booster $\{f^1, \ldots, f^m\}$.[16]

Step $1 \leq t \leq n+1$.

For $k \in \{1, \ldots, m\}$ following the priority orders let any person $i \in I$ *claim* her most preferred object according to P_i among remaining ones in any booster O^k where her priority is $f^k(i) = t$.

In each of the m buckets there are $(n-t)$ objects left. If there are no objects left the algorithm terminates and every person gets assigned her *claimed* objects.

For those interested we next discuss a short example, illustrating our mechanism.

Example 1. Family Heirloom Assignment Problem

Let the set of $n = 3$ individuals, respectively siblings, be $I = \{\text{i, j, k}\}$. The available objects are

[15] A partial order is a *reflexive*, *antisymmetric*, and *transitive* binary relation (Roberts, 1985). We use $O' >_i O''$ to denote that $O' \geq_i O''$ but $O'' \not\geq_i O''$. Similarly we use $O' =_i O''$ whenever $O' \geq_i O''$ and $O'' \geq_i O''$, where given our assumptions on preferences in this case the two sets O' and O'' must be identical. For any $i \in I$ and any ranking $P_i \in \mathcal{P}_i$ respectively $\hat{P}_i \in \mathcal{P}_i$ we will use $\succsim_i$ respectively $\hat{\succsim}_i$ to denote any responsive preference consistent with order P_i respectively $\hat{P}_i$ and $\geq_i$ respectively $\hat{\geq}_i$ for the dominance relation based on P_i respectively $\hat{P}_i$. To reduce notation, we only define the dominance relation for of equal size, which will be sufficient for our purpose.

$O = \{$Armchair, Bagpipe, Clock, Diamond-ring, Earings, Fine wine$\}$ with $n \times m = 2 \times 3$. Moreover, individual i reports P_i : D - B - A - C - F - E, individual j P_j : B - A - C - E - F - D, while individual k reports P_k : B - D - E - F - A - C. Figure 1 illustrates the functioning of the described booster draft algorithm, with boosters $O^1 = \{A, B, C\}$, $O^2 = \{D, E, F\}$, and priority orders $f^1 : i - j - k$, $f^2 : k - j - i$. Even though we have not yet formally introduced the definition, this example portrays a balanced booster draft. It can be easily verified that the final allocation in this case is $A_i = \{B, E\}$, $A_j = \{A, E\}$, and $A_k = \{C, D\}$.

3.2 Properties of the Booster Draft mechanism

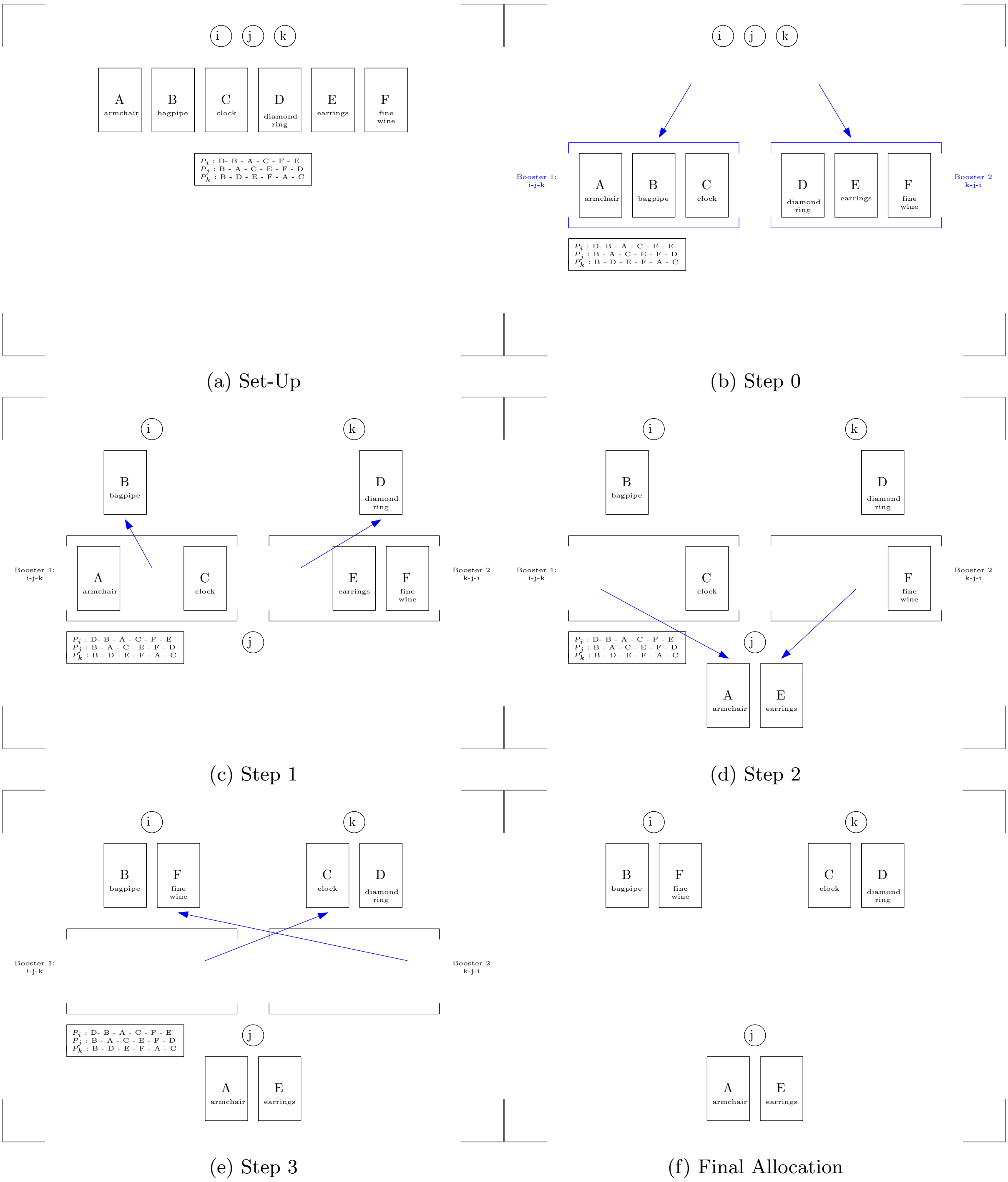
We evaluate mechanisms along three dimensions, whether they are manipulable by submitting untruthful rankings, the efficiency of their outcome, and how fair their assignment is ex post. We start by defining the requirement that individuals should not be able to get a better outcome by misrepresenting their true preferences. A simple mechanism ψ is **strategy-proof** if for all $i \in I$,$P_i, \hat{P}_i \in P_i$, and $P_{-i} \in \mathcal{P}_{-i}$ we have $\psi_i(P) \succsim_i \psi_i(\hat{P}_i, P_{-i})$. Intuitively, we can think of P_i as the truthful report and $\hat{P}_i$ as a possible lie. Strategy-proofness requires that the outcome under a truthful report must be (weakly) preferred to any possible outcome associated with a lie. In order to show that the BD mechanism is strategy-proof we proof the following stronger property: A mechanism ψ is **strongly strategy proof** if for all $i \in I$, $P_i, \hat{P}_i \in \mathcal{P}_i$, and $P_{-i} \in \mathcal{P}_{-i}$ we have $\psi_i(P) \geq_i \psi_i(\hat{P}_i, P_{-i})$. The logic is the same as before, but we require that the outcome is (weakly) preferred under the associated dominance relation $\geq_i$. Together with lemma 1, strong strategy-proofness implies strategy-proofness.

Proposition 1. *The BD mechanism is strongly strategy proof.*

Proof. Suppose by contradiction that there exist $\psi_i(P) \not\geq_i \psi_i(P'_i, P_{-i})$. Then there exists at least one bucket $k \in \{1, \ldots, m\}$ such that $\psi_i^k(P'_i, P_{-1})\ P_i\ \psi_i^k(P)$. But as P_{-i} is fixed all individuals with higher priority will pick identical items in bucket k independent of i reporting P_i or P'_i, so i gets to choose from the same set of remaining objects. Hence we have that the obtained item under P_i is weakly preferred to any item obtained by reporting another simple order, i.e. $\psi_i^k(P)\ R_i\ \psi_i^k(P'_i, P_{-i})$ for all $k \in \{1, \ldots, m\}$ contradicting the initial statement. □

Corollary 1. *The BD mechanism is strategy proof.*

We move on, defining the (ex post) fairness of an outcome. For $j \neq i$ and an outcome of a mechanism $\psi(P) \in \mathcal{A}$ let $\psi(P)^k_{j,i} = \{o \in \psi(P)_j : |\{o' \in \psi(P)_j : o'\ R_i\ o\}| \leq k\}$ denote the set of objects obtained from $\psi(P)_j$ by removing the best k objects according to P_i. Similarly let $\psi(P)^k_{i,i} = \{o \in \psi(P)_i : |\{o' \in \psi(P)_i : o'\ R_i\ o\}| \geq m - k + 1\}$ denote the set obtained by removing the worst k objects from $\psi(P)_i$ following the ranking P_i. We say that mechanism ψ is ***k*-envy free** if for all $P \in \mathcal{P}$ and for all $i, j \in I$ we have



(a) Set-Up
(b) Step 0
(c) Step 1
(d) Step 2
(e) Step 3
(f) Final Allocation

Figure 1: Booster Draft Mechanism

$\psi(P)^k_{i,i} \succsim_i \psi(P)^k_{j,i}$. That is, for any individual i that prefers her bundle to another persons bundle j, we can always remove the best k objects from j's bundle and the k worst objects from i's bundle to eliminate i's envy. Note that if both bundles contain only goods, it would be sufficient to remove only k object from j's bundle to eliminate envy of i. Likewise if both bundles only contain bads we could only remove k objects from i's bundle. By removing both simultaneously we do not need to pay attention whether we remove goods or bads. Naturally, envy-freeness is more demanding the smaller the chosen k. We restrict our attention to a subset of BD mechanism that equalize the priorities across individual as much as possible across the available buckets. That is, for any two individuals $i, j \in I$, we have that i has higher priority than j is at most half of the boosters - rounded up. Formally, a BD mechanism is a **balanced booster draft mechanism** if for all $i, j \in I$ we have $|k \in \{1, \dots, m\} : f^k(i) < f^k(j)\}| \geq \lfloor \frac{m}{2} \rfloor$. Let us next state the trivial observation that the set of balanced booster drafts is always non-empty, followed by proposition 3, stating that balanced booster drafts are $\lceil \frac{m}{2} \rceil$ envy-free.

Proposition 2. *The set of balanced BD rules is non-empty.*

Proof. We simply show this by construction for any m boosters. Fix any priority order f^1. For all $i \in I$ let $f^2(i) = n+1-f^1(i)$, i.e. f^2 reverses the order of priority of f^1. For all odd $k \in \{1, 3, \dots\}$ let $f^k = f^1$ and for all even $k \in \{2, 4, \dots\}$ let $f^k = f^2$. For any $i, j \in I$ if i has a lower priority in all odd (even) priorities than j, i has higher priority than j in all even (odd) priorities, hence it directly follows that $|k \in \{1, \dots, m\} : f^k(i) < f^k(j)\}| \geq \lfloor \frac{m}{2} \rfloor$. □

Proposition 3. *The balanced BD mechanism is* $\lceil \frac{m}{2} \rceil$ *envy-free.*

Proof. Consider the outcome of any balanced booster draft mechanism $\psi(P)$ where some i envies j. Let $K_i = \{k \in \{1, \dots, m\} : f^k(i) < f^k(j)\}$ denote the set of all bucket where i has higher priority than j. Note that every object obtained by i in these buckets must be weakly preferred to any object obtained by j, and hence we have:

$$\bigcup_{k \in K_i} O^k \cap \psi(P)_i \geq_i \bigcup_{k \in K_i} O^k \cap \psi(P)_j$$

Moreover, following P_i, the set obtained by removing the $m - |K_i|$ worst objects from $\psi(P)_i$, denoted by $\psi(P)^{m-|K_i|}_{i,i}$, must weakly dominate the set $\bigcup_{k \in K_i} O^k \cap \psi(P)_i$. Similarly, the set obtained from removing the best $m - |K_i|$, denoted by $\psi(P)^{m-|K_i|}_{j,i}$, objects form $\psi(P)_i$ must be weakly dominated by $\bigcup_{k \in K_i} O^k \cap \psi(P)_j$, and hence:

$$\psi(P)^{m-|K_i|}_{i,i} \geq_i \psi(P)^{m-|K_i|}_{j,i}$$

Note that balancedness implies that $|K_i| \geq \lfloor \frac{m}{2} \rfloor$, i.e. for all $i, j \in I$ we have $|k \in \{1, \dots, m\} : f^k(i) <$

$f^k(j)\}| \geq \lfloor \frac{m}{2} \rfloor$. From this it follows directly that the lower bound on envy for each individual i is $m - |K_i| = m - \lfloor \frac{m}{2} \rfloor = \lceil \frac{m}{2} \rceil$. By lemma 1 we have $\psi(P)_{i,i}^{m-|K_i|} \geq_i \psi(P)_{j,i}^{m-|K_i|}$ implying $\psi(P)_{i,i}^{m-|K_i|} \succsim_i \psi(P)_{j,i}^{m-|K_i|}$, which concludes the proof. □

The last criteria concerns efficiency. We say that a simple mechanism ψ is **Pareto efficient** if for each preference profile $P \in \mathcal{P}$ there does not exist a different allocation $A \in \mathcal{A}$ s.t. everyone prefers the allocation to the outcome under the mechanism, i.e., $A_i \succsim_i \psi(P)_i$ for all $i \in I$ and $A_i \succ_i \psi(P)_i$ for at least some $i \in I$. Following the same logic, we introduce a weaker notion of efficiency, requiring that no allocation can make everyone better off under the dominance relation. A mechanism rule ψ is **dominance efficient** if for each $P \in \mathcal{P}$ there does not exist an allocation $A \in \mathcal{A}$ s.t. $A_i \geq_i \psi(P)_i$ for all $i \in I$ and $A_i >_i \psi(P)_i$ for at least some $i \in I$.

We show that dominance efficiency rules out that any number of individuals can trade single objects with each other and all benefit from the exchange. A **(feasible) single object trade**, under allocation A, is a sequence of individual-object pair $(i_1, o_1), (i_2, o_2), \dots, (i_k, o_k)$ with $o_1 \in A_{i_1}, \dots, o_k \in A_{i_k}$ such that i_2 receives o_1, i_3 receives o_2, so on and so forth, until i_1 receives o_k. An **efficient single object trade** requires that all individual are strictly better off after the trade takes place, i.e., $A_{i_1} \cup \{o_k\} \setminus \{o_1\} \succ_{i_1} A_{i_1}, \dots, A_{i_k} \cup \{o_{k-1}\} \setminus \{o_k\} \succ_{i_k} A_{i_k}$. We show the following characterization result.

Proposition 4. *Under responsive preferences, an allocation A is dominance efficient if and only if there are no efficient single object trades at A.*

Unfortunately, the increased fairness of the BD rule comes at the cost of loosing efficiency, even in its weaker form.

Proposition 5. *The BD mechanism is not dominance efficient.*

Proof. We proceed by counterexample. Let $I = \{1, 2\}$ and $O = \{o_1, o_2, o_3, o_4\}$. Let ψ be the draft mechanism with $O^1 = \{o_2, o_3\}$, $O^2 = \{o_1, o_4\}$ and priority order $f^1 : 1, 2$ and $f^2 : 2, 1$, where we list individuals in order of assigned priority. Suppose the reported ranking is $P_1 : o_1, o_2, o_3, o_4$ and for individual 1 and $P_2 : o_2, o_1, o_4, o_3$ for individual 2. It can easily be checked that the draft mechanism assigns $\psi(P)_1 = \{o_2, o_4\}$ for individual 1 respectively $\psi(P)_2 = \{o_1, o_3\}$ for individual 2. Consider the outcome A obtained by both individuals switching their assignments, i.e. $A_1 = \psi(P)_2$ and $A_2 = \psi(P)_1$. As $A_1 >_1 \psi(P)_1$ and $A_1 >_\psi (P)_2$ the draft mechanism is not dominance efficient. □

As a final remark, we show an impossibility result via counterexample, illustrating that no mechanism can fulfill weak versions of efficiency, fairness and strategy-proofness. We define our weakening of strategy-proofness, allowing only manipulations in which an individual gets at least one object that is strictly better. A mechanism ψ is **dominance strategy proof** if there does not exist $P_{-i} \in \mathcal{P}_{-i}$ and

$P_i, \hat{P}_i \in \mathcal{P}_i$ s.t. $\psi_i(\hat{P}_i, P_{-i}) \geq_i \psi_i(P)$.

Proposition 6. *In the responsive preference domain, there does not exist a simple mechanism that is dominance strategy-proof, dominance efficient, and* $\lceil \frac{m}{2} \rceil$ *envy-free.*

In the appendix, we point out two mechanisms from the literature, one 1-envy free and dominance efficient but manipulable (Harvard business school mechanism) and the other efficient and strategy-proof but k envy-free (serial dictatorship).

4 Partition-Restricted Assignment Domain - Characterization

Intuitively, the arbitrary creation of boosters for the booster draft, leads to a lack of efficiency. Likewise, ex-post fairness suffers from the same problem to a lesser degree. But, if additional information about the specific features of the underlying multi-object assignment problem, is incorporated into the construction of boosters, these issues can be mitigated or even avoided. Of course that only works if there is additional structure to be exploited. Going back to our illustrative family heirloom example, suppose that we have n siblings and $3 \times n$ family heirlooms, consisting of n expensive, n medium priced, and n cheap objects. Moreover, everyone prefers the expensive objects to medium priced, and these to cheap ones, but individuals potentially have different valuations within the three categories. In that case, we get a dominance efficient outcome if the objects are grouped together according to their value, and the outcome of the booster draft is 1-envy free. In this simple example we use additional structure on preferences to build boosters.

For this section, our motivation is based on the assignment of graduate students to teaching positions. Here, students are required to work for exactly one course in each semester, placing a restriction on the allowed allocations. More general, we are given an exogenous partition of objects, in which every individual can be assigned at most one object from every set of the partition.

The **partition-restricted multi-object assignment domain** is a multi object assignment problem $\langle I, O, \succ \rangle$ subject to the constraint that every person can be assigned at most one object from every set O^k for a exogenous give partition $(O^k)_{k \in \{1,\dots,m\}}$. For example, we can think of $\{1, \dots, m\}$ as different time periods for m sets of tasks that have to be carried out, but individuals are not able to work on simultaneously on tasks within the same period. Here, $\succ = (\succ_i)_{i \in I}$ is a list of preferences over **schedules** $S = \{O' \in 2^O : |O^k \cap O'| \leq 1 \text{ for every } k \in \{1, \dots, m\}\}$. A feasible, **restricted allocation** $A = (A_i)_{i \in I}$ is a feasible allocation $A \in \mathcal{A}$ s.t. $A_i \in S$ for all $i \in I$. Let $\mathcal{B}$ denote the set of restricted allocations, clearly $\mathcal{B} \subset \mathcal{A}$. Hence, we refer to this as the partition-restricted assignment domain. For the **partition consistent booster draft**, the m booster are simply $(O^k)_{k \in \{1,\dots,m\}}$. Similarly to before, preferences are responsive if for all $k \in \{1, \dots, m\}$, and $o, o' \in O^k$, as well as $O' \in S$ such that $O' \cap O^k = \varnothing$ we have

$\{o\} \cup O' \succ_i \{o'\} \cup O'$ if and only if $\{o\} \succ_i \{o'\}$.

Given the restriction, individuals no longer need to indicate their preferences across different sets. Therefore, we require every individual $i \in I$ to submit a list of m rankings $P_i = (P_i^1, \dots, P_i^m)$ where P_i^k is a simple order over O^k. For individual $i \in I$, the set of possible messages is $\mathcal{P}_i$. $P = (P_i)_{i \in I}$ is a list of orders for every individual, with the set of all such message profiles being $\mathcal{P}$. We slightly adjust the definition of the dominance relation, i.e. the partial order connecting the reported rankings with the preferences. We say P_i^k is the underlying ranking over O^k if for all $o, o' \in O^k$ we have that $o \; P_i^k \; o'$ if and only if $\{o\} \succ_i \{o'\}$. Let $o'_k = O' \cap O^k$ be the best object simultaneously in subset $O' \in S$ and O^k. For any two subsets $O', O'' \in S$ with $|O'| = |O''| = m$, we have $O' \geq_i O''$ if and only if $o'_k \; R_i \; o''_k$ for all $k \in \{1, \dots, m\}$. The relation between the original preferences $\succsim_i$ of an individual $i \in I$ and the dominance relation $\geq_i$ based on the submitted order P_i remains unchanged. For those interested we moved the exact statement to the appendix.

In the restricted multi-object assignment domain the draft mechanism is dominance efficient. Moreover, we characterize the set of booster draft mechanism as the set of strongly strategy-proof, non-bossy and neutral mechanisms. Formally non-bossiness and neutrality are defined as follows. First, a simple mechanism ψ is **non-bossy** if $\psi_i(P) = \psi_i(\hat{P}_i, P_{-i})$ then $\psi(P) = \psi(\hat{P}_i, P_{-i})$. Secondly, let $\pi : O \to O$ be a permutations s.t. for all $o \in O^k$ we have $\pi(o) \in O^k$. We permute a list of simple orders P, denoted by πP, as follows: For all $k \in \{1, \dots, m\}$ and $o, o \in O^k$ we have $o \; \pi[P_i^k] \; o'$ if and only if $\pi^{-1}[o] \; P_i^k \; \pi^{-1}[o']$. We say a choice rules ψ is **neutral** if for all $k \in \{1, \dots m\}$, for all $i \in I$, and for all possible permutations we have $\pi[\psi(P)_i^k] = \psi(\pi P)_i^k$. Lemma 2 is adapted from Svensson (1999), but we need strong strategy-proofness for the result to go through.

Lemma 2. *Let ψ be a non-bossy and strongly strategy-proof mechanism. Consider $P_i, \hat{P}_i \in \mathcal{P}_i$ and $P_{-i} \in \mathcal{P}_i$. Suppose for all $A_i \in \mathcal{A}_i$ s.t. $\psi_i(P) \geq_i A_i$ we have $\psi_i(P) \hat{\geq}_i A_i$. Then $\psi(P) = \psi(\hat{P}_i, P_{-i})$.*

Proof. By strong strategy-proofness we have $\psi_i(P) \geq_i \psi_i(P'_i, P_i)$.
By the assumption of the lemma we have $\psi_i(P) \hat{\geq}_i \psi_i(P'_i, P_i)$
Using strong strategy-proofness again we get $\psi_i(\hat{P}_i, P_i) \hat{\geq}_i \psi_i(P)$.
Combining the second and third line we get $\psi_i(\hat{P}_i, P_i) = \psi_i(P)$ as this is the only indifference case under the dominance relation $\psi_i(\hat{P}_i, P_i) \hat{=}_i \psi_i(P)$.
By non-bossiness it directly follows that $\psi(P) = \psi(\hat{P}_i, P_{-i})$. □

The set of all **identical preference profiles** is defined as $\mathcal{I} = \{P \in \mathcal{P} : P_j^k = P_i^k$ for all $i, j \in I$ and for all $k \in \{1, \dots, m\}\}$. Considering only identical preference profiles, we show that neutrality strongly restricts the way in which individuals can be assigned objects.

Lemma 3. *Let ψ be a neutral mechanism. For every identical preference profile $P \in \mathcal{I}$, $k \in \{1, \dots, m\}$,* *and* $l \in \{1, \ldots, n\}$ *the same individual* $i_l^k \in I$ *is assigned the* l*th best choice in* O^k *according to preference* P.

Proof. Consider the outcome of a neutral mechanism ψ for any two identical preference profile $P \in \mathcal{I}$ and $\hat{P} \in \mathcal{I}$. Let us define the lth best choice in O^k under the identical preference profile P as well as $\hat{P}$: For all $l \in \{1, \ldots, n\}$ and $k \in \{1, \ldots, m\}$, let o_l^k denote $o \in O^k$ s.t. $|\{o' \in O^k : o' \; R^k \; o\}| = l$ respectively $\hat{o}_l^k$ denote $o \in O^k$ s.t. $|\{o' \in O^k : o' \; \hat{R}^k \; o\}| = l$. Consider the individual i_l^k that is assigned o_l^k under P, i.e. $\psi(P)_{i_l^k}^k = o_l^k$. We want to show that the same individual gets the lth best choice in O^k under any other identical preference profile $\psi(\hat{P})_{i_t^k}^k = \hat{o}_l^k$. Consider the following permutation $\hat{\pi}$ defined for all $k \in \{1, \ldots, m\}$ and $l \in \{1, \ldots, n\}$ as $\hat{\pi}(o_l^k) = \hat{o}_t^k$. For this particular permutation the following holds true:

Claim 1. We have that $\hat{\pi}(P^k) = \hat{P}^k$ for all $k \in \{1, \ldots, m\}$.

Suppose not, then for some $l' < l$ there exists $\hat{o}_{l'}^k \; \hat{P}^k \; \hat{o}_l^k$ such that $\hat{o}_l^k \; \pi[P^k] \; \hat{o}_{l'}^k$. Note that the permuted preference $\hat{o}_l^k \; \pi[P^k] \; \hat{o}_{l'}^k$ is equivalent to the original preference over permuted outcomes $\hat{\pi}^{-1}[\hat{o}_l^k] \; P^k \; \hat{\pi}^{-1}[\hat{o}_{l'}^k]$. But using our defined permutation, this implies $o_l^k \; P^k \; o_{l'}^k$ for $l' < l$ leading to a contradiction.

By neutrality and claim 1 we get $\hat{\pi}[\psi(P)_{i_l^k}^k] = \psi((\hat{\pi}[P]))_{i_t^k}^k = \psi(\hat{P})_{i_t^k}^k$. Moreover by the definition of the permutation $\hat{\pi}$ we have $\hat{\pi}[\psi(P)_{i_l^k}^k] = \hat{\pi}[o_l^k] = \hat{o}_l^k$. Combining both leads the desired conclusion that the same individual gets the lth best object in set O^k for any two identical preference profiles $\psi(\hat{P})_{i_t^k}^k = \hat{o}_l^k$. □

Lemma 3 shows that for identical preference profiles any neutral mechanism can be obtained through a BD mechanism. Note that, in the partition-restricted domain the standard serial dictatorship mechanism is a booster draft where the same individual has the highest priority everywhere, followed by an individual having the second highest priority everywhere and so on and so forth. It remains to be shown, what happens for arbitrary preference profile. We will invoke lemma 2 to show that for any $P \in \mathcal{P} \setminus \mathcal{I}$ there exists an identical preference profile $P \in \mathcal{I}$ leading the same outcome.

Theorem 1. *In the partition-restricted assignment domain with responsive preferences a simple mechanism* ψ *is strongly strategy proof, non-bossy, and neutral if an only if* ψ *is a BD choice rule. The outcome of the BD choice rule is dominance efficient.*

Proof. It is obvious that the BD mechanism is strongly-strategy proof (see proposition 1), neutral and non-bossy.

We now show that any strongly strategy proof, non-bossy, and neutral simple mechanism ψ is booster draft. Apply ψ to the subset of identical preference profiles $\mathcal{I}$. By lemma 3 for each $k \in \{1, \ldots, m\}$ and $l \in \{1, \ldots, n\}$ we can uniquely identify an individual $i_l^k \in I$ that is assigned her t-th choice in O^k according to preference P. Formally the outcome of the BD mechanism is defined for all $k \in \{1, \dots, m\}$ recursively from highest to lowest priority individuals as $\psi_i^k(P) = \{o \in O^k : o \; R_i \; o'$ for all $o' \in O^k \setminus \bigcup_{j \in \{j \in I: f^k(j) < f^k(i)\}} \psi_j^k(P)\}$. Indeed assigning the individuals priorities in the same order they obtain the objects from each booster we get that $\psi_i^k(P) = o_t^k$ for all $l \in \{1, \dots, n\}$ and $k \in \{1, \dots, m\}$. Therefore for each identical preference profile $\mathcal{I}$ the outcome of any neutral mechanism ψ is obtained by a booster draft mechanism.

It remains to be shown for any other preference profile that is not identical across agents. Consider a preference profile $\hat{P} \in \mathcal{P} \setminus \mathcal{I}$ and construct an identical preference profile $P(\hat{(P)}) \in \mathcal{I}$ from it as follows: For any O^k with $k \in \{1, \dots m\}$ the preference P^k order ranks individual i_1^1's first choice highest, and individuals i_1^t first choice among the remaining objects in O^k as t-th highest for $t \in \{2, \dots, n\}$.

By lemma 2 we can move all agents preferences one-by-one from the constructed identical preference profile $P(\hat{(P)})$ back to the initial preference profile $\hat{P}$ without changing the outcome of the mechanism. Hence for any preference profile the outcome of any neutral, strongly strategy-poof and non-bossy mechanism is a booster draft mechanism.

Finally we show that the outcome of the booster draft mechanism in this domain dominance efficient. Suppose by contradiction there exists $A_i \geq_i \psi(P)_i$ for all $i \in I$ holding strictly for at least one individual. $A_i \geq_i \psi(P)_i$ means that $A_i^k \; R_i^k \; \psi(P)_i$ for all $k \in \{1, \dots, m\}$ and for all $i \in I$ holding stickily for at least some k and i. If there are multiple, pick the first basket k and the first agents that gets a strictly better object in k. As all agents with higher priority in O^k get the same items as before A_i^k is still available and therefore we have $\psi(P)_i^k \; R_i^k \; A_i^k$ contradicting $A_i^k \; P_i^k \; \psi(P)_i$.

□

The subset of balanced booster draft mechanism is $\frac{m}{2}$ envy-free. As a robustness check for the theorem, note that the result no longer holds in the unrestricted multi-object assignment problem.

As mentioned, in the partition-restricted assignment domain the serial dictatorship mechanism is a special case of the (non-balanced) BD mechanism, which is not true for the responsive domain. So the result doesn't go through for the unrestricted problem, as there exists a mechanism that is strongly strategy-proof, non-bossy and neutral outside the set of BD mechanism.

5 Teaching Assignments for Graduate Students

In this section, we take a closer look at the teaching assignment problem, which is an example of the previously described partition-restricted assignment domain. The data consists of the rankings submitted by graduate students in economics at Boston College for the academic year 2018, as well as the final assignment made for that year. Analogous to our theoretical part, everyone separately ranked the available positions for each semester and was assigned a single teaching position for both the fall and spring semester.[17] We simulate the outcome of the balanced booster draft (**BD**) as well as the serial dictatorship (**SD**) for 10000 different priority orders, while the actual assignment (**AA**) remains unchanged. As in the theoretical part we want to analyze the different assignments in terms of efficiency and fairness.

For fairness, we care about the percentage of students envying at least one other student, who was given a strictly better assignment in both semesters (2 Envy). For completeness, we also show the percentage of remaining students envying at least on student for his or her assignment in one semester (1 Envy), and the percentage of students getting their two top choices (Envy 0). Figure 2 shows that the balanced booster draft mechanism avoids "2 Envy" altogether, and therefore outperforms serial dictatorship where "2 Envy" is roughly 15%, as well as the actual assignment where "2 Envy" reaches almost 30%.

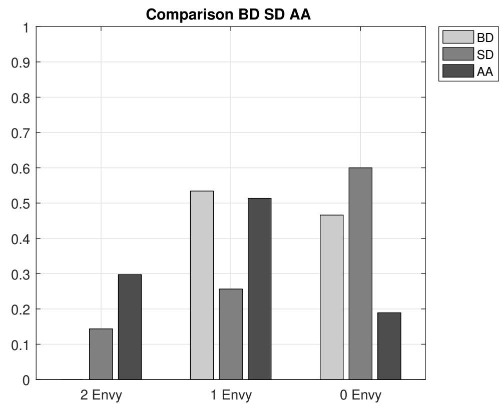


Figure 2: Envy

For efficiency figure 3 depict the percentage of individuals that get at least one strictly better and one weakly better assignment in both semesters under one outcome relative to another. The actual assignment in that year is unsatisfactory in terms of efficiency compared to the two alternatives. On the other hand there is no noticeable difference between serial dictatorship and the balanced booster draft mechanism.

Finally, following Budish and Cantillon (2012), we consider the average rank, i.e., a simple measure of welfare to compare the tree alternatives. For example, if a graduate student is assigned her first choice in one semester and her third choice in the other, her rank is four. We then simply average across all

[17] For each semester, graduate students give their preferences over ta (teaching assistant) principles, ta statistics, ta econometrics, lab (laboratory) stats, lab econometrics, tf (teaching fellow) principles, and tf statistics. In our data 5 out of 37 students made special arrangements with a specific professor or got a fellowship that freed them of work for one semester. In those cases, we always assigned them their pre-arragned positions before assigning positions to the remaining students based on their reported rankings. As the was a new person in charge of the assignment for 2018 and it was unknown how reported rankings would translate into the final assignment, it is reasonable to expect that graduate students reported their rankings truthfully.

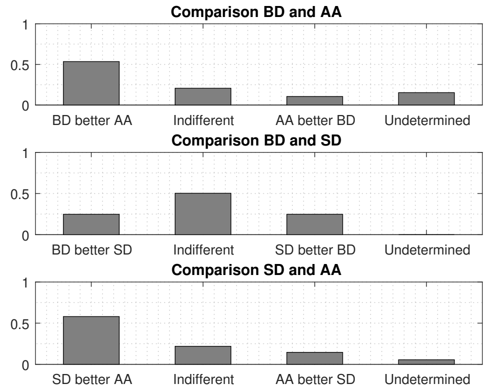


Figure 3: Efficiency

students for a given final allocation. Again the results are consistent with the previous analysis in that both serial dictatorship and the booster draft have an average rank of 3.21, while the average rank for the actual assignment is 4.72. Moreover, the balanced booster draft leads to a lower dispersion in terms of rank compared the serial dictatorship.

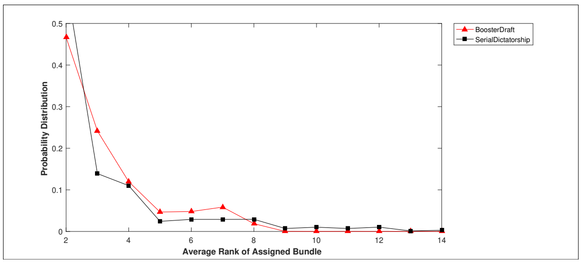


Figure 4: Distribution Average Rank

We conclude that if individuals are mildly risk-avers their will prefer the (random) balanced booster draft to the (random) serial dictatorship.

6 Dominance Obvious S-P

Following the idea of obviously strategy-proofness introduced by Li (2017), we give some insight in which sense the BD mechanism can be implemented as a extensive form game that is easy to understand. In other words, we think an additional strength of the booster draft lies the mechanisms simplicity. This turns out to be important in practice, as mentioned in the literature review.

As in the previously, I is the set of individuals, $\mathcal{A}$ is the set feasible allocations, and each individual i has a preference relation $\succsim_i$ over the outcomes, which we will sometimes refer to as the **type** of an agent. Preferences are responsive, and assignments are either made in the partition-restricted or the unrestricted domain. A type profile $\succsim = (\succsim_i)_{i \in I}$ specifies a preference relation for each person, and the set of all type profiles is denoted by $\succsim^I$.

Consider an extensive game form where each terminal history z results in some outcome $g(z) \in \mathcal{A}$. For ease of presentation we focus on the special case of deterministic games with finite preference, finite outcomes sets, and complete information.[18] $\mathcal{G}$ denotes the set of all such game forms, with representative element G. Table 1 depicts useful notation.

Name	Notation	Representative Element
histories	H	h
initial history	$h_{\varnothing}$	
terminal histories	Z	z
outcome resulting from z	$g(z)$	
individual called to play at h	$i(h)$	
information sets for agent i	$\mathcal{I}_i$	I_i
actions available at I_i	$A(I_i)$	$a(I_i)$

Table 1: Notation Extensive Form Games

A **strategy** S_i for agent i chooses an action $S_i(I_i) \in A(I_i)$ at every information set. A strategy profile $S = (S_i)_{i \in I}$ specifies a strategy for each agent, and $\mathcal{S}$ denotes the set of all strategy profiles. A **type-strategy profile function** $T : \succsim^I \to \mathcal{S}$ specifies a strategy profile for every type profile. Any persons type-strategy depends only on her own type. $T(\succsim_i) \in \mathcal{S}_i$ refers to the strategy assigned to type $\succsim_i$. Let $z^G(h, S)$ be the terminal histories that results in game form G when starting from h and play proceeding according to S.

For a given extensive form game G and a particular type $\succsim_i$, strategy S_i is **weakly dominant** if $\forall S_i'$ and $\forall S_{-i}$ we have $g(z^G(h_{\varnothing}, S_i, S_{-i})) \succsim_i g(z^G(h_{\varnothing}, S_i, S_{-i}))$. Similarly we can define the stronger requirement of an obviously strategy-proof strategy profile. For this we first need to introduce some additional notation. For two distinct strategies S_i and S_i', an information set is in the set of **earliest points of departure** $I_i \in \alpha(S_i, S_i')$ if it is on the path of play under both S_i and S_i', and both strategies choose the same action at all earlier information sets but select a different action at I_i. Furthermore let $Z^G(I_i, S_i)$ denote the set of **reachable terminal histories** by playing strategy S_i when starting from information set I_i in game G. Given G and $\succsim_i$, S_i is **obviously dominant** if $\forall S_i'$ and $\forall I_i \in \alpha(S_i, S_i')$ there does not exist $z' \in Z^G(I_i, S_i')$ and $z \in Z^G(I_i, S_i)$ such that $g(z') \succsim_i g(z)$.[19]

A **mechanism** is a function $\psi : \succsim^I \to \mathcal{A}$ from type profiles to assignments. A **solution concept** $C(\cdot)$ maps any game G into a subset of strategy profiles $C(G) \subseteq \mathcal{S}$ satisfying the solution concept C. An extensive form game together with a type strategy profile function (G, T) is said to **C-implement** a mechanism ψ if $\forall \succsim \in \succsim^I$ we have $T(\succsim) \in C(G)$ as well as $\psi(\succsim) = g(z^G(h_{\varnothing}, T(\succsim)))$. Similarly ψ is C-implementable if there exist (G, T) that satisfy the above requirements. In particular a type-strategy profile $T(\succsim) \in SP(G)$ is in the set of **strategy proof (SP)** profiles if for all $i \in I$, $T(\succsim_i)$ is weakly dominant. A type-strategy profile $T(\succsim) \in OSP(G)$ is in the set of **obviously strategy-proof (OSP)**

[18] In a complete information games every information set is a singleton.

[19] Unlike the original definition, our version of obvious dominance is slightly modified, allowing to consider preferences that do not compare all available options in $\mathcal{A}$.

profiles if for all $i \in I$, $T(\succsim_i)$ is obviously dominant. Unfortunately the booster draft mechanism is not implementable in a obvious strategy-proof way.

Proposition 7. *In the restricted and unrestricted assignment domain with $m \geq 2$, $n \geq 2$ the balanced BD mechanism is not OSP implementable.*

We weaken the concept of obvious strategy-proofness, by specifying a subset of pairwise comparisons between outcomes, an individual "pays attention to". The more comparisons can be made, the closer the definition is to standard OSP. Here we focus on our dominance relation, though we provide a more general definition, for an arbitrary partial order, in the appendix. Given G and $(\succsim_i, \geq)$, S_i is **dominance obviously dominant** if $\forall S_i'$ and $\forall I_i \in \alpha(S_i, S_i')$ there does not exist $z' \in Z^G(I_i, S_i')$ and $z \in Z^G(I_i, S_i)$ such that $g(z') >_i g(z)$. A type-strategy profile $T(\succsim) \in DOSP(G)$ is in the set of **dominance obviously strategy-proof (DOSP)** profiles if for all $i \in I$, $T(\succsim_i)$ is dominance obviously dominant.

Proposition 8. *In the unrestricted domain BD is not DOSP implementable. In the partition-restricted assignment domian the balanced BD mechanism is DOSP implementable.*

Returning to our original motivation, we note that the extensive form game specified by the card version of the booster draft mechanism actually obviously dominance strategy-proof implements the BD mechanism. In other words it provides us with an additional explanation why drafting rules might be easy to understand pointing to their prevalence in practice.

7 Conclusion

We have introduced the booster draft mechanism, a new allocation scheme for the multi-object assignment problem, inspired by existing drafting procedures in competitive card games. In the responsive preference domain, the BD mechanism is strategy-proof and envy-free equal to half of the objects, but it is neither dominant efficient nor dominance obvious strategy-proof implementable. In the partition-restricted assignment domain, any neutral, non-bossy, and strongly strategy-proof simple mechanism is a BD mechanism. Moreover, the subset of balanced BD is dominance efficient, strategy-proof, envy-free equal to half of the object, and dominance obvious strategy-proof implementable. We discuss a practical application in the partition-restricted assignment domain, the assignment of graduate students to teaching assistant positions. The simulated assignments support the claims made in the theoretical argument of the paper.

A Mathematical Appendix

A.1 Section 3

A.1.1 Proposition 4: Characterization Dominance Efficiency

Proof. We start with the if-statement. Suppose that it does not hold, then the allocation A is dominance efficient, but there exists a efficient single object trade. One can easily confirm that, under responsive preferences a single object trade $(i_1, o_1), (i_2, o_2), \dots, (i_k, o_k)$ makes every individual involved $\{i_i, \dots, i_k\}$ better off under the dominance relation, i.e., $A_{i_1} \cup \{o_k\} \setminus \{o_1\} >_{i_1} A_{i_1}, \dots, A_{i_k} \cup \{o_{k-1}\} \setminus \{o_k\} >_{i_k} A_{i_k}$. Hence carrying out the trade makes all individual in the trade strictly better of under the dominance relation while everyone else is indifferent, contradicting that A is dominance efficient.

Next consider the only-if-statement. Suppose it is does not hold, then there exists no efficient single object trade at A, but A is not dominance efficient. Let A' be an allocation that is dominant efficient relative to A and consider the following argument.

Step 0. Pick any individual i for which $A' >_i A$ and call it i_k. Pick the best object $o \in A'_{i_k} \setminus A_{i_k}$ and call it o_{k-1}. This object must have been assigned to a different person under A. Call that person i_{k-1} and go to the next step.

Step t. Consider individual i_{k-t}. Pick the best object $o \in A'_{i_{k-t}} \setminus A_{i_{k-t}}$ and call it o_{k-t-1}. This object must have been assigned to a different person under A. If the individual is in $\{i_{k-t+1}, \dots, i_k\}$ we found a efficient single object trade, starting from the original distribution A, and hence reach a contradiction. Otherwise call that person i_{k-t-1} and go to the next step.

As the set of individuals is finite we reach a contradiction after a finite number of steps. Every person in the circle get his/her best object among new ones. □

A.1.2 Proposition 6: Impossibility Result

Counterexample. Let $I = \{i, j\}$ and $O = \{o_1, o_2, o_3, o_4\}$. Suppose ψ is a dominance strategy-proof, dominance efficient and $\lceil \frac{m}{2} \rceil = 1$ envy-free mechanism. We abbreviate rankings P_i as 1234_i to represent $o_1\ P_i\ o_2\ P_i\ o_3\ P_i\ o_4$. Similarly for an allocation A with $A_i = \{1, 2\}$ and $A_j = \{3, 4\}$ we simply write $(12, 34)$.

Case 1: $\psi(1234_i, 1234_j) = (12, 34)$

This outcome violates $\lceil \frac{m}{2} \rceil = 1$ pick envy freeness. We have that j envies i's assignment as $\{12\} >_j \{34\}$ and even after removing the best object from i and the worst form j the envy prevails as $\{1, 2\} \setminus \{1\} >_j \{3, 4\} \setminus \{4\}$, respectively $\{2\} >_j \{3\}$.

Case 2: $\psi(1234_i, 1234_j) = (34, 12)$

The reasoning is symmetric to the one in case 1.

Case 3: $\psi(1234_i, 1234_j) = (24, 13)$

Observation 1: $\psi(1234_i, 3214_j) = (14, 23)$. Individual j can always change her preference from 3214_j to 1234_j and get $\{1, 3\}$. This leaves us already with only two possible outcomes of the mechanism either $(24, 13)$ or $(14, 23)$. Given dominance efficiency we must have $\psi(1234_i, 3214_j) = (14, 23)$ as $\{1, 4\} \geq_i \{2, 4\}$ and $\{2, 3\} \geq_j \{1, 3\}$.

Observation 2: $\psi(1234_i, 2341_j) = (14, 23)$. By observation 1, any other outcome would violate dominance strategy proofness as j can switch back to 3214_j and enforce her best outcome $\{2, 3\}$.

Observation 3: $\psi(2314_i, 1234_j) \neq (34, 12)$. This follows by dominance strategy profness at otherwise i would be better of under preferences consistent with 2314_i to report 1234_i instead and get $\{2, 4\}$ instead of $\{3, 4\}$.

Observation 4: $\psi(2314_i, 2341_j)$ has no outcome is consistent with the required criteria.

- Naturally $(14, 23)$ and $(23, 14)$ are both violating 1 envy freeness, as i respectively j get their worst two objects.
- By dominance strategy proofness we can rule out $(34, 12)$. For this we need to note that by observation 3 we have $\psi(2314_i, 1234_j) \neq (34, 12)$, and therefore fixing i's ranking no other preference of j can ever give her the outcome $\{1, 2\}$.
- Using observation 2 and the same logic we can rule out $(12, 34)$ and $(13, 24)$ as $\psi(1234_i, 2341_j) = (14, 23)$, fixing j's ranking we can never have that i gets $\{1, 3\}$ or $\{1, 2\}$.
- This leaves us with $\psi(2314_a, 2341_b) = (24, 13)$. Consider $(12, 34)$ which can be reached by letting i and j trade o_1 and o_4 making both strictly better off and hence violating dominance efficiency.

This leads to the conclusion that no dominance strategy-proof, dominance efficient, and 1 pick envy free can every assign $\psi(1234_a, 1234_b) = (24, 13)$.

Case 4: $\psi(1234_i, 1234_j) = (13, 24)$
We can symmetrically follow the previous reasoning of case 3.

Case 5: $\psi(1234_a, 1234_b) = (23, 14)$

Observation 1: $\psi(2341_a, 1234_b) = (23, 14)$. This simply follows from dominance strategy proofness as under 2341_a we have $(23, 14) = \psi(1234_a, 1234_b) >_a \psi(2341_a, 1234_b) \neq (23, 14)$.

Observation 2: $\psi(2341_a, 2314_b)$ has no outcome is consistent with the required criteria.

- Outcomes $(14, 23)$ and $(23, 14)$ are both violating 1 envy freeness.

- Outcome $(12, 34)$ is dominated by $(24, 13)$ similarly $(13, 24)$ is dominated by $(34, 12)$, hence both would violate dominance efficiency.
- The last two possible outcomes violate dominance strategy proofness as under 1234_j we have $(34, 12)$ or $(24, 13) = \psi(2341_a, 2314_b) >_j \psi(2341_a, 1234_b) = (23, 14)$.

This leads to the conclusion that no dominance strategy-proof, dominance efficient, and $\lceil \frac{m}{2} \rceil = 1$ envy free mechanism can every assign $\psi(1234_a, 1234_b) = (23, 14)$.

Case 6: $\psi(1234_a, 1234_b) = (14, 23)$
We can symmetrically follow the previous reasoning of case 5.

Case 1-6 together conclude the proof as regardless of what we assign $\psi(1234_a, 1234_b)$ we find a contradiction with at least one required property.

□

A.1.3 Serial Dictatorship and Harvard Business School Mechanism

Fixing a single priority order serial dictatorship algorithm is formally defined by the following algorithm:

SD Algorithm

Step $1 \geq t \leq n$.

There are $m \times n - m \times (t-1)$ objects left. Following the priority orders let person $i \in I$ with priority $f^{-1}(i) = t$ pick her k most preferred objects among the remaining objects.

Similarly we can define the Harvard Business School mechanism via the following algorithm. This algorithm is based on two priority orders f^{odd} and f^{even} that have reverse priority, i.e. $f^{odd}(i) = n - f^{even}(i)$.

HBS Algorithm

Step $1 \geq t \leq n \times m$.

There are $m \times n - (t-1)$ objects left. The priority order used is changed all n steps from f^{odd} to f^{even} and back. Following the appropriate priority order $f \in \{f^{odd}, f^{even}\}$ let person $i \in I$ with priority $f^{-1}(i) = t$ pick her most preferred object among the remaining objects.

It is well know that serial dictatorship is efficient and strategy-proof, but is unsatisfactory in terms of ex-post fairness. The HBS algorithm on the other hand is 1 envy-freeness and dominance efficiency but fails even the weaker notion of dominance strategy proofness. We summarize these results for the responsive preference domain in the following propositions.

A.1.4 Proposition 9: Efficiency

Proposition 9. *HBS is dominance efficient. SD is pareto efficient (among all outcomes giving each individual exactly m objects) and hence dominance efficient. BD is not dominance efficient.*

Proof. HBS is dominance efficient. Let $\psi(P)$ be the outcome of the HBS mechanism. Suppose to the contrary that there exists an assignment A s.t. $A_i \geq_i \psi_i(P)$ for all $i \in I$ and $A_i >_i \psi_i(P)$ for at least some $i \in I$, It is obvious that when ordering the objects in $\psi_i(P)$ following P_i we get that the object in lth place $\psi_i^l = \{o \in \psi_i(P) : |\{o' \in \psi_i(P) : o' \; R_i \; o\}| = l\}$ for $l \in \{1, \ldots, k\}$ is the lth object picked under the HBS mechanism. As $A_i \geq_i \psi_i(P)$ for every object ψ_i^l assigned at each step of the HBS mechanism there exists an object $o_i^l \; R_i \; \psi_i^l$ with $o_i^l = \{o \in A_i : |\{o' \in A_i : o' \; R_i \; o\}| = l\}$. Consider the first step of the HBS mechanism where $o_i^l \; P_i \; \psi_i^l$. Note that as P is a simple order, all previous objects must have been identical $o_i^l = \psi_i^l$. This leads to a contradiction as the object ψ_i^l assigned under the HBS mechanism must be the best available object following P_i but there exists $o_i^l \; P_i \; \psi_i^l$.

HBS is not pareto efficient. Let $I = \{1, 2\}$ and $O = \{o_1, o_2, o_3, o_4\}$. Suppose the reported ranking is $P_i : o_1, o_2, o_3, o_4$ for $i = 1, 2$. Under $f^{odd} : 1, 2$ and $f^{even} : 2, 1$, the outcome under the HBS mechanism is $\psi_1(P) = \{o_1, o_4\}$ and $\psi_2(P) = \{o_2, o_3\}$. Note that preferences $\succsim_1$: $\{o_2, o_3\}, \{o_1, o_4\}$ and $\succsim_2$: $\{o_1, o_4\}, \{o_2, o_3\}$ are both consistent with the reported order P_1 respectively P_2 as the relative ranking between the two bundles cannot be inferred from the reported simple order under responsive preferences. Therefore assignment A with $A_1 = \{o_2, o_3\}$ and $A_2 = \{o_1, o_4\}$ pareto dominates the outcome $\psi(P)$.

SD is pareto efficient. It is well known that serial dictatorship is efficient and therefore dominance efficient. Note that the highest priority person i_1 with $f^{-1}(i_1) = 1$ gets her m best objects. Under responsiveness of P_{i_1} the bundle containing the best m objects is the best set in $\{O' \in 2^O : |O'| = m\}$. Conditional on this the second highest priority person i_2 with $f^{-1}(i_2) = 2$ gets her best m objects among the remaining object. As we can never change i_1's assignment to another assignment containing m objects without making her worse off we can never changes i_2's assignment as well. Following this argument for the remaining individual we can conclude that there cannot exist an allocation assigning every person weakly better bundle of size m.

SD is not pareto efficient under any assignment. Let $I = \{1, 2\}$ and $O = \{o_1, o_2, o_3, o_4\}$. Suppose the reported ranking is $P_i : o_1, o_2, o_3, o_4$ for $i = 1, 2$. Suppose that for 1 we have that $\{o_2, o_3, o_4\} \succ_1 \{o_1, o_2\}$ while for 2 we have $\{o_1\} \succ_2 \{o_2, o_3, o_4\}$.

For example u_1 and u_2 are additive utility functions of individual 1 and 2 where $u_1 : 100, 99, 98, 88$ and $u_2 : 100, 3, 2, 1$. Where $u_1(\{o_2, o_3, o_4\}) = 285 > u_1(\{o_1, o_2\}) = 199$ and $u_2(\{o_1\}) = 100 > u_1(\{o_2, o_3, o_4\}) = 6$. Under priority order $f : 1, 2$ the outcome of m-serial dictatorship is $\psi(P)$ with $\psi(P)_1 = \{o_1, o_2\}$ with $\psi(P)_2 = \{o_3, o_4\}$ which is Pareto dominated by A with $A_1 = \{o_2, o_3, o_4\}$ with $A_2 = \{o_1\}$. $\square$

A.1.5 Proposition 10: Envy Freeness

Proposition 10. *HBS is* 1 *envy-free. BD is* $\lceil \frac{m}{2} \rceil$ *envy-free. And SD is* m*-pick envy free.*

Proof. HBS. 1 envy freeness follows directly from the algorithm. Consider any pair of individuals $i, j \in I$. Under the reported preference profile P_i person i always prefers her first picked item to person j's second picked item, her second picked item to person j's third picked item, and so on and so forth. Hence under any allocation of the draft mechanism we get that that $o_i^k \; P_i \; o_j^{k+1}$ for $l \in \{1, \dots, m-1\}$ which implies 1 envy-free.

SD. Let ψ denote the serial dictatorship mechanism. We have that $|\psi_i(P)| = |\psi(P)_j| = m$ for any two assignments. Consider two individual with identical simple orders. It follows that the one with lower priority will have m objects all worse that the higher priority individual. Hence by removing m items from any two sets we end up with $\varnothing \geq_i \varnothing$. □

A.1.6 Proposition 11: Strategy Proofness

Proposition 11. *BD and SD are both strongly strategy-proof. The HBS mechanism is not dominance strategy proof.*

Proof. HBS. Let $I = \{1, 2\}$ and $O = \{o_1, o_2, o_3, o_4\}$. Suppose true preference are $P_1 : o_1, o_2, o_3, o_4$ for individual 1 respectively $P_2 : o_2, o_3, o_4, o_1$ for individual 2. Take priority orders $f^{odd} : 1, 2$ and $f^{even} : 2, 1$, then under the HBS mechanism ψ and true rankings we get $\psi_1(P_1, P_2) = \{o_2, o_3\}$ and $\psi_2(P_1, P_2) = \{o_1, o_4\}$. But there is a profitable manipulation for 1 by picking the more popular object first $\hat{P}_1 : o_2, o_1, o_3, o_4$ leading $\psi_1(\hat{P}_1, P_2) = \{o_1, o_2\}$ and $\psi_2(\hat{P}_1, P_2) = \{o_3, o_4\}$. Its easy to check that $\psi_1(\hat{P}_1, P_2) >_1 \psi_1(\hat{P}_1, P_2)$ and hence violating even dominance strategy proofness.

SD is strategy-proof. Let ϕ denote the serial dictatorship mechanism. Is is well known that the serial dictatorship mechanism is strategy proof. The highest priority agent $i_1 \in I$ with $f^{-1}(i_1) = 1$ obtains the best m objects. Under responsiveness of P_{i_1} the bundle containing the best m objects is the best set in $\{O' \in 2^O : |O'| = m\}$. As any outcome for i_1 under the serial dictatorship is in the set $\{O' \in 2^O : |O'| = m\}$ we have $\psi(P)_{i_1} \succsim_i \psi(P_{i_1}, P'_{i_1})_{i_1}$ for all $P_{i_1}, P'_{i_1} \in \mathcal{P}_{i_1}$. The second highest priority agent $i_2 \in I$ with $f^{-1}(i_2) = 2$ obtains the best m object among the remaining ones and hence can never be better off by misrepresenting using an analogous argument Following this argument step by step for every individual leads us to the desired conclusion.

□

A.2 Section 4

The following is a restatement of lemma 1, adjusted to the partition-responsive domain.

Lemma 4. *Let $\succsim_i$ be any responsive preference relation over 2^O with underlying ranking P_i, and $\geq_i$ the corresponding dominance relation. For any $O', O'' \in S$ if $O' \geq_i O''$ then $O' \succsim_i O''$.*

Proof. Suppose we have $O', O'' \in S$ with $O' = \{o'_1, \ldots, o'_m\} \geq_i O'' = \{o''_1, \ldots, o''_m\}$. As $O' \geq_i O''$ we have $o'_1 \ R_1 \ o''_1$ as well as $\{o'_1\} \succsim_i \{o''_1\}$. Using responsiveness for $\{o''_2, \ldots, o''_m\} \cap \{o'_1, o''_1\} = \varnothing$ and $\{o'_1\} \succsim_i \{o''_1\}$ we get $\{o'_1, o''_2, \ldots, o''_m\} \geq_i O'' = \{o''_1, o''_2 \ldots, o''_{m'}\}$. Replacing one-by-one o''_k by o'_k for all $k \in \{2, \ldots, m\}$ and invoking responsiveness we get $O' = \{o'_1, o'_2, \ldots, o'_m\} \succsim_i \ldots \succsim_i \{o'_1, o''_2, \ldots, o''_{m'}\} \succsim_i \{o''_1, o''_2, \ldots, o''_{m'}\} = O''$. By transitivity of $\succsim_i$ we reach the conclusion that $O' \succsim_i O''$. □

A.3 Section 7

A.3.1 Proposition 7: Not OSP

Proof. Following proposition 2 (pruning principle) in Li (2017), we can restrict attention to "minimal" extensive form games, where no histories are off the path of play. It is also sufficient to show that a sub-function ψ is not OSP-implementable. Take agents $\{1, 2\} \subseteq I$ and objects $\{a, b, c, d\} \subseteq O$ with $\{a, b\} \subseteq O^1$ and $\{c, d\} \subseteq O^2$ with priority orders $f^1(1) < f^1(2)$ and $f^2(2) < f^2(1)$. Consider the following subset $\succsim^{\{1,2\}} \subset \succsim^N$ for the partition-restricted assignment domain.

$$\succsim_1\colon \{a, c\} \succ_1 \{b, c\} \succ_1 \{a, d\} \succ_1 \{b, d\} \tag{1}$$

$$\succsim_2\colon \{a, c\} \succ_2 \{a, d\} \succ_2 \{b, c\} \succ_2 \{b, d\} \tag{2}$$

Take any G pruned with respect to the truthful strategy profiles, such that G OSP-implements ψ for domain $\succsim^{\{1,2\}}$. Consider some history h at which $i(h) = 1$ with a non-singleton action set. If OSP holds this cannot come before any non-singleton action history with $i(h) = 2$. Suppose not, then if 1 chooses an action corresponding to $a \ P_a \ b$ her worst payoff is $\{a, d\}$ while her best payoff under an action corresponding to $b \ P_a \ a$ is $\{b, c\} \succ_1 \{a, d\}$. Similarly consider some history h at which $i(h) = 2$ with a non-singleton action set. This cannot come before any such history with $i(h) = 1$. Suppose not, then if 2 chooses an action corresponding to $c \ P_a \ d$ her worst payoff is $\{b, c\}$ while her best payoff under an action corresponding to $d \ P_a \ d$ is $\{a, d\} \succ_2 \{b, c\}$. So all action sets of 1 and 2 must be singletons and G does not OSP-implement the BD rule. □

A.3.2 Partial Order SP, OSP, WGSP

Slightly abusing notation, a **partial order function** $\trianglerighteq : \succsim^I \to \trianglerighteq^I$ is a consistent way to assign a subset of pairwise comparisons $\trianglerighteq(\succsim_i) \subseteq \succsim_i$ to each type. Given G and $(\succsim_i, \trianglerighteq)$, S_i is **partial order obviously dominant with respect to** $\trianglerighteq$ if $\forall S'_i$ and $\forall I_i \in \alpha(S_i, S'_i)$ there does not exist $z' \in Z^G(I_i, S'_i)$ and $z \in Z^G(I_i, S_i)$ such that $g(z') \triangleright(\succsim_i) \ g(z)$. A type-strategy profile $T(\succsim) \in OSP_{\trianglerighteq}(G)$ is in the set of

partial order obviously strategy-proof (PoOSP) profiles if for all $i \in I$, $T(\succsim_i)$ is partial order obviously dominant with respect to $\trianglerighteq$.

Similarly we can define the standard concepts of strategy proofness and weak group strategy proofness for a particular partial order. For the partial order profile $\geq = (\geq_i)_{i \in I}$ a type-strategy profile $T(\succsim) \in SP_{\trianglerighteq}(G)$ is in the set of **partial order strategy-proof (PoSP)** profiles if there does not exists an $\succsim \in \succsim^I$, individual $i \in I$ with deviation strategy $\hat{S}_i \neq T(\succsim_i)$ such that $g(z^G(h_\varnothing, \hat{S}_i, T(\succsim) \setminus T(\succsim_i))) \; \triangleright(\succsim_i) \; g(z^G(h_\varnothing, T(\succsim)))$. For the partial order profile $\geq = (\geq_i)_{i \in I}$ a type-strategy profile $T(\succsim) \in WGSP_{\trianglerighteq}(G)$ is in the set of **partial order weakly group-strategy-proof (PoWGSP)** profiles if there does not exists a coalition $I' \subseteq I$, type profile $\succsim \in \succsim^I$, deviating strategies $\hat{S} = (\hat{S}_i)_{i \in I'}$, non coalition strategies $T(\succsim) \setminus \hat{S}$ such that for all $i \in I'$ we have $g(z^G(h_\varnothing, \hat{S}, T(\succsim) \setminus \hat{S})) \triangleright(\succsim_i) \, g(z^G(h_\varnothing, T(\succsim)))$.

Proposition 12. *If $T(\succsim) \in OSP_{\trianglerighteq}(G)$ with respect to $\trianglerighteq$ then $T(\succsim) \in WGSP_{\trianglerighteq'}(G)$ with respect to the same partial order $\trianglerighteq' = \trianglerighteq$.*

Proof. Suppose $T(\succsim) \notin WGSP_{\trianglerighteq}(G)$. Then there exists coalition I' with types $(\succsim_i)_{i \in I'}$ that can deviate to strategy $\hat{S} = (\hat{S}_i)_{i \in I'}$ where all $i \in I$ are strictly better off following the partial order $\geq$. Along the resulting terminal history, there is a first agent $i \in I'$ in the coalition to deviate from $T(\succsim_i)$ to $\hat{S}_i$. That first deviation happens at some information set $I_i \in \alpha(T(\succsim_i), \hat{S}_i)$. We have for $h \in I_i$ and $S'_{-i} = (T(\succsim) \setminus \hat{S}) \cup (\hat{S} \setminus \{\hat{S}_i\})$ that

$$g(z^G(h, \hat{S}_i, S'_{-i})) \triangleright(\succsim_i) \, g(z^G(h, T(\succsim)))$$

We reach a contradiction as $T(\succsim) \notin OSP_{\trianglerighteq}(G)$ as there exists an earliest point of departure at which a preferred history following the partial order is reachable. □

A.3.3 Proposition 8: DOSP

We fist show that in the standard domain the balanced BD mechanism is not DOSP implementable. We show that the mechanism is not DWGSP(G) implementable, and hence by proposition 12, in the previous appendix subsection, not DOSP implementable.

Proposition 13. *In the responsive preference domian for $m \geq 2$ the balanced BD mechanism is not DWGSP(G) implementable.*

Proof. Let $I = \{1, 2\}$ and $O = \{o_1, o_2, o_3, o_4\}$ without loss of generality let the partition of objects into packages be $O^1 = \{o_1, o_2\}$ and $O^2 = \{o_3, o_4\}$ as well as $f^1(1) > f^1(2)$ and $f^1(2) > f^1(1)$. Suppose agent

1's type $\succsim_1$ produces the following simple order preference $P_1 : o_3, o_1, o_2, o_4$ respectively agent 2's type the simple order $P_2 : o_1, o_3, o_4, o_2$.
The BD mechanism leads $A_1 = \{o_1, o_4\}$ for individual 1 and $A_2 = \{o_2, o_3\}$ for individual 2.
Now consider the manipulation corresponding to $\hat{P}_1 :: o_3, o_2, o_1, o_4$ and $\hat{P}_3 :: o_4, o_1, o_2, o_3$. The BD mechanism leads $A_1 = \{o_2, o_3\}$ for individual 1 and $A_2 = \{o_1, o_4\}$ for individual 2. This is strictly better for both 1 and 2 following the dominance relation as for individual 1 we have $o_3\ P_1\ o_1$, $o_2\ P_1\ o_4$ and while for individual 2 we have $o_1\ P_2\ o_3$, $o_4\ P_2\ o_2$. □

Corollary 2. *In the responsive preference domain, if ψ is a balanced BD mechanism and $m \geq 2$, then there does not exist $G \in \mathcal{G}$ that DOSP-implements ψ.*

We now show the second part, i.e. that in the restricted domain the Balanced BD is DOSP implementable.

Note that the BD rule is a simple mechanism that treats all types with the same underlying simple order as identical. We will think of the problem as having a mechanism from $\psi : \mathcal{P} \to \mathcal{A}$ and specifying a type-strategy profile function $T : \mathcal{P} \to \mathcal{S}$.

Proof. Define the *extensive form game* G as follows. Take the buckets $\{O^1, \dots, O^m\}$ and the corresponding priority orders $\{f_1, \dots, f_m\}$. The set of players is I. The set of actions at each information set $A(I_i)$ is to claim a single available object. For histories of length $|h| \in [1, n-1]$ the player at each node is identified by the priority order f_1. Similarly for any history of length $|h| \in [kn, (k+1)n - 1]$ the player $P(h)$ is defined by the priority order f_k. The terminal histories give every agents the set of object the person claimed at each step of the path leading to z.
Define the *type strategy* $T(P_i)$ for each type $\succ_i$ corresponds to a simple order P_i over individual objects. In particular $T(P_i)$ is simply to take the best object available at each information set where a agent is called to play following the simple order P_i. Its straight forward to see that $\psi(P) = g(z^G(h_\varnothing, T(P)))$ where ψ is the outcome of the BD mechanism.

We want to show that $T \in DOSP(G)$ for all i and for all $\succsim_i$, $T_i(\succsim_i)$ is obviously dominance relation dominant. Suppose to the contrary that there exists $i \in I$ such that for some S_i' and $I_i \in \alpha(S_i, S_i')$ there exists $z' \in Z^G(I_i, S_i')$ and $z \in Z^G(I_i, S_i)$ with $g(z') \geq (P_i)\ g(z)$. So for all $k \in \{1, \dots m\}$ we have that $g(z')_i \cap O^k\ R_i\ g(z)_i \cap O^k$. This implies that at I_i the object obtained under the two strategies differs $S_i(I_i) \neq S_i'(I_i)$ where $S_i'(I_i), S_i'(I_i) \in O^{k\prime} \subseteq O^k$ for some $k \in \{1, \dots m\}$. But we know that under the type strategy $S_i(I_i)\ R_i\ S_i'(I_i)$ holding strictly as $S_i(I_i) \neq S_i'(I_i)$, i.e. $S_i(I_i)\ P_i\ S_i'(I_i)$ contradicting that the assignment under S_i' dominates the assignment under S_i as $g(z)_i \cap O^k\ P_i\ g(z)_i \cap O^k$. □

B Illustration Algorithms

We schematically depict the described algorithms for eight object and four individual. In line with the original inspiration, the objects are represented by cards. Moreover, we invite the observer to interpret the depicted square as a tabletop with the individual 1,2,3, and 4 sitting around that table.

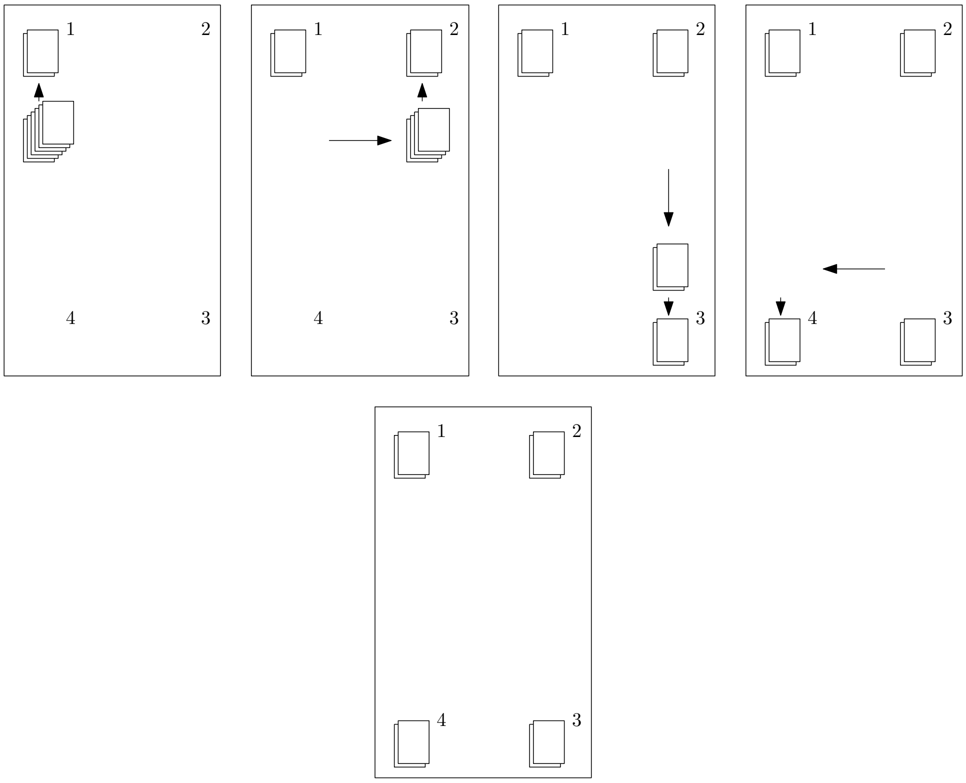


Figure 5: Schematic Depiction Serial Dictatorship

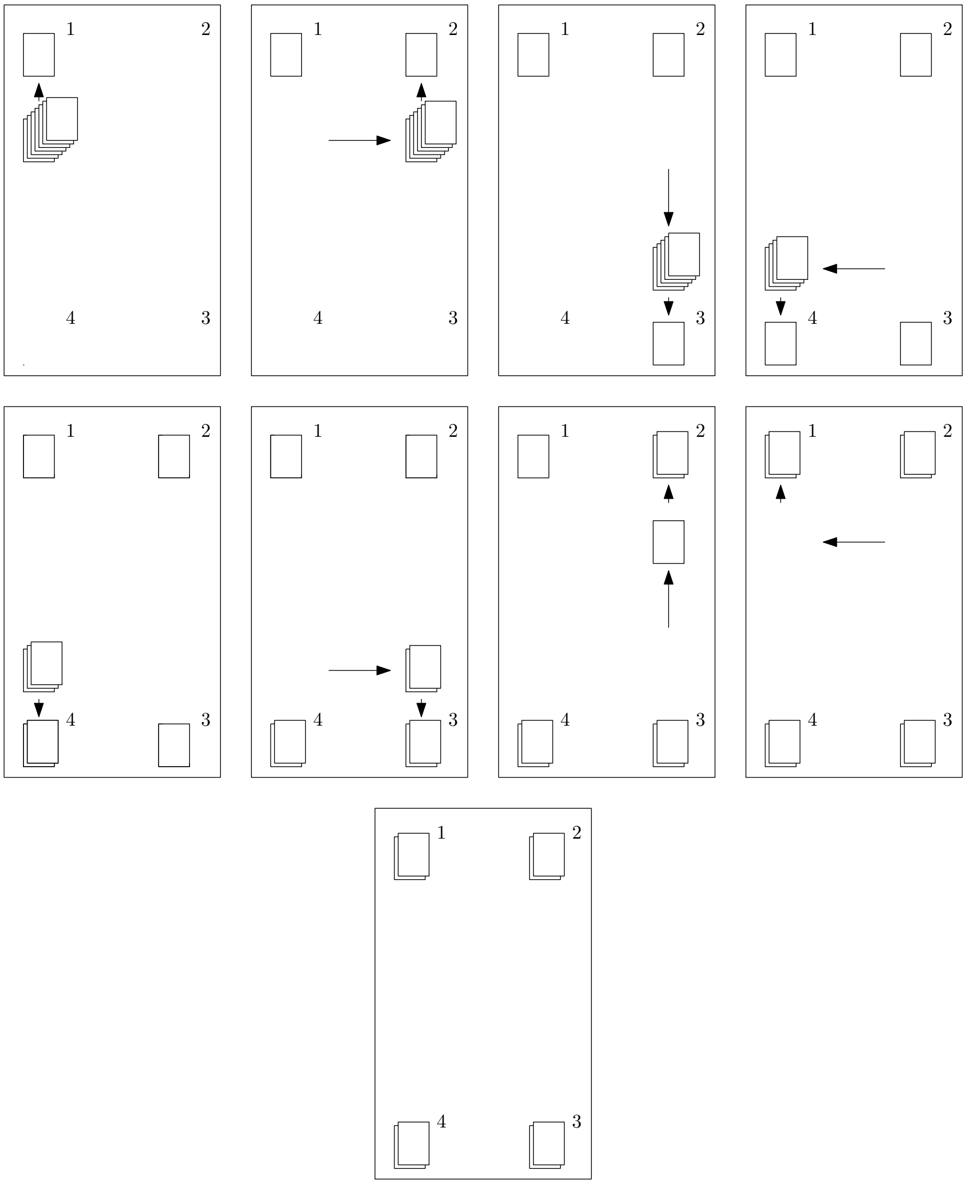


Figure 6: Schematic Depiction Harvard Business School Mechanism

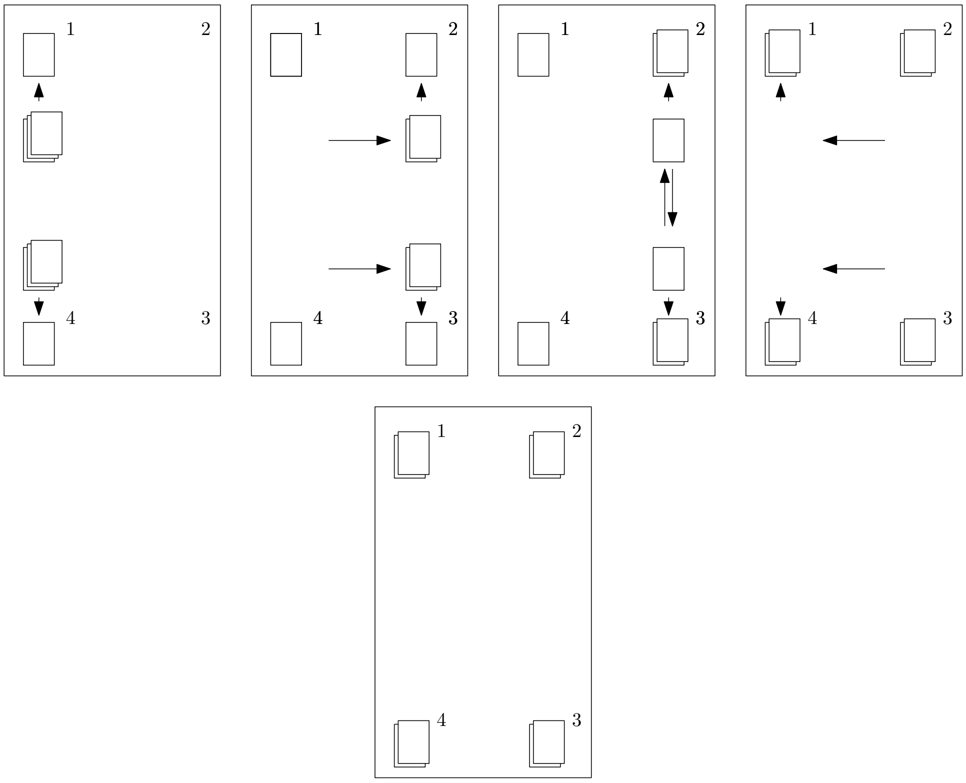


Figure 7: Schematic Depiction Booster Draft Mechanism

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