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# DISCUSSION PAPER

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## Commitment in First-Price Auctions

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## Abstract

We study the role of commitment in a first-price auction environment. We devise a simple two-stage model in which bidders first submit an initial offer that the auctioneer can observe and then make a counteroffer. There is no commitment on the auctioneer's side to accept an offer as is or even to choose the lowest bidder. We compare this setting to a standard first-price auction both theoretically and experimentally. While theory suggests that the offers and the auctioneer's revenue should be higher in a standard first-price auction compared to the first-price auction with renegotiation, we cannot confirm these hypotheses in the experiment.

**Keywords:** Auctions; Experiment.

**JEL Classification:** D44; D47.

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# 1 Introduction

The question whether to commit to clear rules when selecting the winner plays a large role in most real-life procurement processes. The multi-attribute nature of the goods or services to be procured makes a binding price-only auction a suboptimal choice. In this type of auction, the buyer cannot account for factors that she deems relevant for her awarding decision in the auction itself. From her perspective, a non-binding negotiation format where she chooses the winner after having seen all the offers might seem attractive. This non-commitment to rules on how a winner is chosen allows for flexibility when taking other, non-price attributes, into account. To support this, Jap (2002) points out that many auctions in procurement are carried out in a non-binding fashion.

This paper investigates the role of commitment in a concise setting and examines whether participants react to commitment, or a lack thereof, in a first-price auction. We compare a standard first-price auction with commitment to a first-price auction where renegotiation is possible, while varying as little as possible between the two settings. In our simple two-stage mechanism, bidders first submit an offer that the auctioneer can observe. In the second stage, the auctioneer then selects a winner and can make a counteroffer. There is no commitment on the auctioneer's side to accept an offer as is or to choose the lowest bidder.<sup>1</sup> In theory and considering that the auctioneer makes a counteroffer, this means that bidders pool on bids that reveal no information about their costs. This means, in equilibrium, bids are uninformative and the auctioneer implements the ex-ante optimal take-it-or-leave-it offer.

We then take these mechanisms into the laboratory where we benchmark the theoretical model of step two against a standard first-price auction. Contrary to theoretical predictions, we observe no significant difference in the offers between the setting with renegotiation and the standard first-price auction. Also, we find evidence that first-stage offers are correlated to the private information of the bidders in both settings.

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<sup>1</sup>Even if the auctioneer did commit to choosing the lowest offer, the offers would still be uninformative.

There is evidence in the literature that having a binding auction, or an auction with commitment, is an important factor when designing the procurement process. The most related study was conducted by Fugger, Katok, and Wambach (2016). They show that conducting auctions without commitment can lead to non-competitive prices. In their study, a quality component is introduced that is unknown to the auctioneer before the auction. The authors then compare two settings of a dynamic reverse auction: with and without commitment. The auctioneer conducts either a price-only auction, where the lowest bid wins, or a buyer-determined auction. In the latter, she chooses the winner after having seen all the offers and qualities. Since bidders do not know their quality ranking, they cannot be sure that a reduction in price leads to a higher winning probability. Therefore, bidders lack an incentive to submit competitive offers and collusion on high prices prevails. They show theoretically that these non-competitive offers become profitable once the auctioneer does not commit to clear rules on how the winner is chosen. This theoretical finding is then confirmed via a laboratory experiment. Our study is focussed on keeping the mechanism as simple as possible to isolate the role of commitment. There exists one type of equilibria in both our settings with clear predictions, collusion is not profitable. While offers in the standard first-price auction are competitive, theory predicts that bidders pool on offers that reveal no information about their type in the first-price auction with renegotiation.

Commitment has been studied mainly in the multi-attribute literature and the optimal mechanism-design without commitment literature. Che (1993) analyzes the role of commitment in multi-attribute auctions. If the auctioneer is able to commit to a scoring rule, then the optimal scoring rule undervalues quality with respect to the auctioneer's utility. If not, the only scoring rule she can implement is given by her utility. In contrast to this paper, their perspective is to derive optimal buyer behavior in the presence and absence of commitment power. They also theoretically show the importance and benefits of commitment. We, on the other hand, focus on bidder behavior in settings with and without commitment.

In the optimal mechanism-design literature Vartiainen (2013) shows that if a sequentially rational auctioneer cannot commit to the mechanism rules, the only

mechanism she can implement is a variant of the English auction. Mechanisms in which offers directly depend on a bidder's type are generally not possible. The English auction has the property that the winner of the auction does not reveal his offer (and type), while in our first-price auction, this is not the case. Also, in our paper, the auctioneer cannot choose the procurement mechanism and is bound, depending on the setting, to either a standard first-price auction or a first-price auction with renegotiation. McAfee and Vincent (1997) assume more structure. The auctioneer sets a reserve price but cannot commit to not reauction the good if the reserve-price is not met. They show that in this case, the revenue of the auction drops to the static auction without reserve price. This is related to our setting, where in the first-price auction with renegotiation, the buyer can enforce a reserve-price via take-it-or-leave-it offer if the offers do not meet her expectations. This is possible because the buyer still wields some commitment power, namely that reauctioning is not possible. Once the chosen bidder has declined the counteroffer, no deal is made.

Related to commitment in auctions is Tan (1996). The author studies a procurement setting where a buyer is privately informed about her own demand. If the buyer is able to commit a reserve price, it is always in her interest to do so. This means that she reveals her private information. In our setting, the auctioneer does not possess private information. Also, communication is only possible from the suppliers to the auctioneer in the form of offers.

The paper is organized as follows. In section 2, we develop the model and analyze it. In section 3, we describe our experimental setting and present the results.

## 2 Model

In this section, we introduce the formal model. We consider an auctioneer and  $n$  bidders that compete for one indivisible good in a two-stage mechanism.<sup>2</sup> We

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<sup>2</sup>We write our model as a selling rather than a procurement mechanism, since our experiment is framed as a selling auction, too. This has the advantage that we have consistency in notation throughout the paper. This is, of course, without loss of generality.

assume that both bidders and the auctioneer are risk-neutral profit maximizers.

Bidders' values for the good are independently and identically distributed according to a cumulative distribution function  $F$  over the set  $V = \{v, \dots, \bar{v}\}$ ,  $v \geq 0$  and  $V \subset \mathbb{N}^0$ .<sup>3</sup> The auctioneer assigns zero value to the good.

In the first stage, bidders send an offer to the auctioneer. Offers are binding, the auctioneer may acquire the good for any offer that was submitted. The set of possible offers is given by  $B = \mathbb{N}^0$ .

In the second stage, the two settings we compare differ. In the first-price auction with renegotiation, there is no commitment on the auctioneer's side. She observes the offers and can choose the winner arbitrarily. She then makes a counteroffer to the chosen bidder or accepts the offer as-is. The bidder can accept or decline the counteroffer. In the standard first-price auction, the auctioneer observes the offers and chooses one of them.

Utilities are identical in both settings. For the auctioneer, her utility is given by the price paid by the winner of the auction if the trade takes place. The utility of the chosen bidder with value  $v$  winning with a price of  $p$  is given by

$$u_b(v; p) = v - p. \tag{1}$$

The bidder that was not chosen has a utility of zero. If no trade takes place, the utility of everyone is zero.

## 2.1 Analysis

We show that there exists a continuum of equilibria in the no-commitment setting. Each equilibrium is characterized by bidders mixing over a subset  $U_o \subset B$  such that  $\max\{U_o\} \leq \underline{v}$ . Note that  $U_o$  may contain only one element,  $p_o$ . In that case, bidders pool on  $p_o$ . For  $\underline{v} = 0$ , the equilibrium is unique.

**Proposition 1.** *The equilibria are characterized by*

1. *Bidders: randomize over a subset  $U_o \subset B$  such that  $\max\{U_o\} \leq \underline{v}$  in the first stage*

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<sup>3</sup>The exact spacing between types and bids is not important, as long as the spacing in the bid and type spaces stays constant.

2. Auctioneer:

- i) observes that all offers are  $\in U_o$ : she chooses a bidder at random and makes a counteroffer. The counteroffer  $p_{co}$  is equal to the ex-ante optimal take-it-or-leave-it offer,  $p_{co} = \arg \max_{p \in \{\underline{v}, \dots, \bar{v}\}} (1 - F(p))p$ .
- ii) observes one or multiple offers  $\notin U_o$ : she chooses a deviating offer and makes a counteroffer that is equal to  $\bar{v}$ .

We start by showing that the proposed behavior indeed forms an equilibrium. Bids are binding, so every bidder submitting offers above his value has an incentive to deviate to a lower offer. This means for any value larger than  $\underline{v}$ , there is a non-zero possibility that the bidder cannot make that offer. This means bidders cannot pool on any value larger than  $\underline{v}$  and it follows that bidders pool by mixing over a subset  $U_o \subset B$  such that  $\max\{U_o\} \leq \underline{v}$ . Off-equilibrium beliefs of the auctioneer are given by  $\mu(v_i = \bar{v} | o_i \notin U_o) = 1$ , meaning that if she observes any signal  $\notin U_o$  in the first stage, she assumes that the bidder is of the highest type. Hence, deviating always yields a revenue of zero for the bidder, he receives a counteroffer of  $\bar{v}$ . The intuition behind these off-equilibrium beliefs comes from how an auctioneer would eliminate possible types.

Suppose there are  $n + 1$  types,  $V = \{0, 1, \dots, \bar{v}\}$  and let  $0 < p_{co} < \bar{v}$ . If the auctioneer observes an offer of 1, this bidder must be of type  $v \in \{1, \dots, \bar{v}\}$ . Bidders of type  $v = 1$  can send offers of 0 or 1. But the smallest counteroffer an auctioneer could commit to after observing 1 would be 1. This means bidders of value  $v = 1$  are indifferent and submit only offers of 0. Bidders of type  $v = 2$  can send offers of 0, 1 or 2. Since bidders of type  $v = 1$  do not send offers of 1, the smallest counteroffer an auctioneer could commit to after observing 1 or 2, would be 2. Therefore, the expected profit of a bidder of type  $v = 2$  who submits an offer of 1 or 2 is 0 and  $\frac{1}{2} \max\{2 - p_{co}, 0\}$  if he submits an offer of 0. It follows that, like the type- $v = 1$  bidders, bidders of type  $v = 2$  will only submit offers of 0. This argument can be chained  $n$  times until only the bidder of type  $\bar{v}$  is left. The best response of the auctioneer facing these uninformative offers is setting the optimal reserve price,  $p_{co} = \arg \max_{p \in \{\underline{v}, \dots, \bar{v}\}} (1 - F(p))p$ .

We will now show that this is the unique type of equilibrium. Bidders are assumed to be profit maximizers. This means they will accept any counteroffer that is larger or equal than their value. Assume the bidders submit offers according to a separating equilibrium bidding function.<sup>4</sup> A separating equilibrium bidding function implies that the auctioneer can infer the value of each bidder from the offer. She would then make a counteroffer to the bidder with the highest and extract full surplus in the second stage equal to his value for the good. The bidder would then accept this counteroffer and make a profit of zero. This means bidders would always prefer to imitate a lower type. This rules out the existence of any separating equilibrium. The same logic can be applied over any subset of  $V$ .

It is left to show that there exist no partial pooling equilibria where bidders pool on multiple offers in  $V \setminus \{v\}$ . Consider a setting with  $m$  pooling offers  $s_i \in V \setminus \{v\}$  with  $i \in \{1, 2, \dots, m\}$ . W.l.o.g., let  $v_i \in V$  be the lowest type that sends  $s_i$ . Let  $p_i, i \in \{1, 2, \dots, m\}$ , be the respective prices an auctioneer sets after observing that the highest bid is  $s_i$ . W.l.o.g., let  $p_1 < p_2 < \dots < p_m$ . The auctioneer cannot commit to any  $p_i \leq v_i$  since she knows that the lowest type sending the signal  $s_i$  has a value of  $v_i$ . On the other hand, a bidder having a value of  $v_i$  will never send a signal that results in a price  $p \geq v_i$  since deviating to a lower signal would earn him a strictly positive expected payoff. This is a contradiction to the assumption that  $v_i$  is the lowest type sending signal  $s_i$ , meaning that no pooling equilibrium with one or multiple offers in  $V \setminus \{v\}$  can exist.

In the standard first-price auction, bidders send an offer to the auctioneer in the first stage. The set of possible offers is the same as before,  $B = \mathbb{N}_0$ . The auctioneer then observes these offers and has to choose one of the offers. She cannot make a counteroffer. This setting is equivalent to a first-price auction: A profit maximizing auctioneer will always select the highest offer. First-price auctions are well-studied in the literature, see for example Krishna (2009), for the discrete case see Chwe (1989) and Cai, Wurman, and Gong (2010). The equilibrium bidding function for a bidder with value  $v$  bidding against  $n - 1$  other bidders is approximated well by the continuous equilibrium bidding function if there are a sufficient number of bid

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<sup>4</sup>This equilibrium bidding function does not need to be monotone.



steps,

$$\beta^I(v) = \frac{1}{F_1^{(n-1)}(v)} \int_0^v y f_1^{(n-1)}(y) dy. \quad (2)$$

**Proposition 2.** *Bids in the standard first-price auction are higher or equal than in the first-price auction with renegotiation.*

**Proposition 3.** *The standard first-price auction is more efficient than the first-price auction with renegotiation is not.*

From Chwe (1989) and Cai, Wurman, and Gong (2010), we know that for any type, bids are higher or equal than  $\underline{v}$ . This is in contrast to the first-price auction with renegotiation where every submitted offer is smaller or equal to  $\underline{v}$ , proving Proposition 2. In the standard first-price auction, the good is always sold in equilibrium. In the first-price auction with renegotiation, the bidder might reject the counteroffer, making this format inefficient. This proves Proposition 3.

## 2.2 Quantal Response Equilibrium

The predictions for the first-price auction with renegotiation are extreme in the sense that for any deviation from the equilibrium, the auctioneer's counteroffer jumps from the optimal take-it-or-leave-it offer to  $\bar{v}$ . Bidders are assumed to perfectly understand that the auctioneer can infer their type in any separating bidding strategy and that their bid should not contain any information about their type. In comparison to the standard first-price auction, where small errors only lead to small changes in winning probability and expected payment, the first-price auction with renegotiation leaves no room for errors. Still, in real-life situations, bidders and the auctioneer might err due to, for example, cognitive limitations. In this section, we are interested in what happens when we relax the assumption that players' choices are always optimal and allow them to make mistakes.

One equilibrium concept choice to account for these type of deviations is the quantal response equilibrium (QRE) (McKelvey and Palfrey, 1995). In this section we model both the first-price auction with renegotiation and the standard first-price auction settings and derive the corresponding response functions. We begin with the first-price auction with renegotiation. Consider  $n = 2$  bidders. Let  $T =$

$\{0, \dots, 10\}$  be the set of possible types.<sup>5</sup> The action space of the bidders is given by  $A^B = T = \{t_i\}_{i \in \{0, \dots, 10\}}$ . The auctioneer's action space is given by

$$A^A = \{(t_0, b^1), (t_1, b^1), \dots, (t_{10}, b^1), (t_0, b^2), \dots, (t_{10}, b^2)\} = \{a_i^A\}_{i \in \{0, \dots, 21\}}, \quad (3)$$

where the first eleven entries denote a counteroffer of  $t_i$  to bidder one while the other entries denote the counteroffers to bidder two. Note that both offers and counteroffers are capped by the highest possible type. In QRE, every action of every player is chosen with a positive probability depending on the expected utility of said action and on a precision parameter,  $\lambda \in [0, \infty)$ . We use the logit QRE concept described in Goeree, Holt, and Palfrey (2016) in chapter 3.3.

Consider bidder 1. Let  $\sigma_{ij}^B$  be the probability that a bidder of type  $t_i$  submits an offer of  $t_j$ . Let  $\sigma_{ijk}^A$  be the probability that, given the bids of bidder one and two,  $b^1 = t_i$  and  $b^2 = t_j$ , the auctioneer chooses the action  $a_k^A$ . The weighting function depends on the expected utilities. Then the expected utility of bidder 1 being of type  $t_i$  and submitting an offer of  $t_j$  is given by

$$U_1^B(t_i, t_j, \sigma^B, \sigma^A) = \underbrace{\sum_{k=0}^{10} \sum_{l=0}^k \sigma_{kl}^B}_{\text{bidder 2: } b^2} \underbrace{\sum_{m=0}^{21} \sigma_{jlm}^A}_{\text{action: } a_m^A} \begin{cases} t_i - t_m & a_m^A \in (\cdot, b^1) \text{ \& } m \leq i \\ 0 & a_m^A \notin (\cdot, b^1) \\ 0 & a_m^A \in (\cdot, b^1) \text{ \& } m > i \end{cases} \quad (4)$$

$$= \sum_{k=0}^{10} \sum_{l=0}^k \sigma_{kl}^B \sum_{m=0}^i \sigma_{jlm}^A (t_i - t_m) \quad (5)$$

$$:= U_{1,ij}^B(\sigma^B, \sigma^A). \quad (6)$$

Analogously, the expected utility of the second bidder being of type  $t_i$  and submit-

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<sup>5</sup>We consider a reduced version with eleven types of the experiment that has 101 types. This is due to computational limitations when numerically solving the QRE.

ting an offer of  $t_j$  is given by

$$U_2^B(t_i, \sigma^B, \sigma^A) = \sum_{k=0}^{10} \sum_{l=0}^k \sigma_{kl}^B \sum_{m=0}^{21} \sigma_{jlm}^A \begin{cases} t_i - t_m & a_m^A \in (\cdot, b^2) \text{ \& } m \leq i + 11 \\ 0 & a_m^A \notin (\cdot, b^2) \\ 0 & a_m^A \in (\cdot, b^2) \text{ \& } m > i + 11 \end{cases} \quad (7)$$

$$= \sum_{k=0}^{10} \sum_{l=0}^k \sigma_{kl}^B \sum_{m=11}^{i+11} \sigma_{jlm}^A (t_i - t_{m-11}) \quad (8)$$

$$:= U_{2,ijk}^B(\sigma^B, \sigma^A). \quad (9)$$

The expected utility of the auctioneer having received the offers  $b^1 = t_i$  and  $b^2 = t_j$  and taking action  $a_k^A$  is given by

$$U_{ijk}^A(\sigma^B, \sigma^A) = \begin{cases} \sum_{m=0}^{10} \sigma_{mi}^B \begin{cases} t_k & m \geq k \\ 0 & m < k \end{cases} & a_k \in (\cdot, b^1) \\ \sum_{m=0}^{10} \sigma_{mj}^B \begin{cases} t_k & m + 11 \geq k \\ 0 & m + 11 < k \end{cases} & a_k \in (\cdot, b^2) \end{cases} \quad (10)$$

$$= \begin{cases} \sum_{m=k}^{10} \sigma_{mi}^B t_k & a_k \in (\cdot, b^1) \\ \sum_{m=k-11}^{10} \sigma_{mj}^B t_k & a_k \in (\cdot, b^2). \end{cases} \quad (11)$$

The logit QRE response function to determine the  $\sigma$ 's in the quantal response equilibrium is generally of the form

$$\sigma_i = \frac{e^{\lambda U(\sigma_i)}}{\sum_{\sigma_j} e^{\lambda U(\sigma_j)}}. \quad (12)$$

In our case, we have the following system of equations,

$$\begin{aligned} \sigma_{ij}^B &= \frac{e^{\lambda U_{1,ij}^B(\sigma^B, \sigma^A)}}{\sum_{k=0}^i e^{\lambda U_{1,ik}^B(\sigma^B, \sigma^A)}} & \forall t_i, t_j \in T \\ \sigma_{ij}^B &= \frac{e^{\lambda U_{2,ij}^B(\sigma^B, \sigma^A)}}{\sum_{k=0}^i e^{\lambda U_{2,ik}^B(\sigma^B, \sigma^A)}} & \forall t_i, t_j \in T \\ \sigma_{ijk}^A &= \frac{e^{\lambda U_{ijk}^A(\sigma^B, \sigma^A)}}{\sum_{m=0}^{21} e^{\lambda U_{ijm}^A(\sigma^B, \sigma^A)}} & \forall t_i, t_j \in T \text{ and } \forall a_k^A \in A^A. \end{aligned} \quad (13)$$

We make some assumptions on the behavior of both the bidders and the auctioneer.

The bidders cannot submit bids strictly higher than their type, so  $\sigma_{ij}^B = 0$  for all  $j > i$ . The auctioneer takes this into account and forgoes strictly dominated choices when submitting the counteroffer. Therefore, she does not make counteroffers lower than the highest of offers she has received. This means  $\sigma_{ijk}^A = 0$  for all  $k < \max\{i, j\}$  and  $11 < k < \max\{i, j\} + 11$ . Additionally, we assume that the auctioneer chooses the highest of the two bidders for the counteroffer,  $\sigma_{ijk}^A = 0$  for  $k < 12$  if  $i < j$  and  $\sigma_{ijk}^A = 0$  for  $k > 11$  if  $i > j$ . While this assumption increases the pressure on prices, it does not change the results qualitatively and makes the presentation of the results easier. This is due to the fact that the probability of an action  $a_k^A$  then depends only on the highest bid, which yields a probability matrix that is easier to interpret. The standard first-price auction is modeled analogously, see Appendix 5.2.

In Goeree, Holt, and Palfrey (2016) it is shown that for  $\lambda \rightarrow \infty$ , the QRE converges to the unique Bayes-Nash-equilibrium derived in the last section. This means QRE gives us three predictions for the behavior of bidders and auctioneer:

**Proposition 4.** *For the limit case  $\lambda \rightarrow \infty$ , bids are lower in the first-price auction with renegotiation than in the standard first-price auction.*

As shown in Goeree, Holt, and Palfrey (2016) section 3, the logit QRE converges to the unique Bayes-Nash-equilibrium derived in the last section. Then the results derived in that section apply.

**Proposition 5.** *For the limit case  $\lambda \rightarrow 0$ , bids are identical in the first-price auction with renegotiation and the standard first-price auction. Bidders are unresponsive to expected payoffs and submit all valid offers with equal probability.*

For  $\lambda \rightarrow 0$ , the system (13) simplifies to

$$\begin{aligned}\sigma_{ij}^B &= \frac{1}{i+1} & \forall t_i, t_j \in T \\ \sigma_{ij}^B &= \frac{1}{i+1} & \forall t_i, t_j \in T \\ \sigma_{ijk}^A &= \frac{1}{22} & \forall t_i, t_j \in T \text{ and } \forall a_k^A \in A^A.\end{aligned}\tag{14}$$

**Proposition 6.** *In contrast to the unique Bayes-Nash-equilibrium derived in the last section, for any  $\lambda > 0$ , there is correlation between the offers submitted by the bidders and the counteroffer submitted by the auctioneer.*

For  $\lambda > 0$ , bidders submit all offers larger than zero and smaller or equal than their type with strictly positive probability. The auctioneer then conditions her counteroffer on the bids she received and forgoes strictly dominated actions, namely those counteroffers smaller than the highest of offers. This means that there exists a correlation between the offers and the counteroffer.

We can numerically compute the equilibrium probability weights as described in Goeree, Holt, and Palfrey (2016) for different  $\lambda$  values. The results can be found in Figure 1 – Figure 3.

The QRE of the standard first-price auction can be found in Figure 1. As expected, for the higher  $\lambda$ -value, the offers are less “washed out” around the standard equilibrium bidding strategy of around  $v/2$ .

The QRE of the first-price auction with renegotiation can be found in Figure 2 and Figure 3. For the bidders, one can still see some pressure to pool offers in the  $\lambda = 15$  case, while in the more error-prone  $\lambda = 3$  case, offers start resembling those of the first-price auction. For the counteroffers, the auctioneer makes use of the information she gets from the bidders and mixes her response.

In conclusion, we might observe offers in the first-price auction with renegotiation that are closer to the offers in the standard first-price auction than standard theory would predict.

### 3 Experiment

In this section, we introduce our experimental design and state our hypotheses for the experiment.

#### 3.1 Experimental Design

We conducted three different treatments: the standard first-price auction (FPA), the first-price auction with renegotiation (FPR) and the first-price auction with

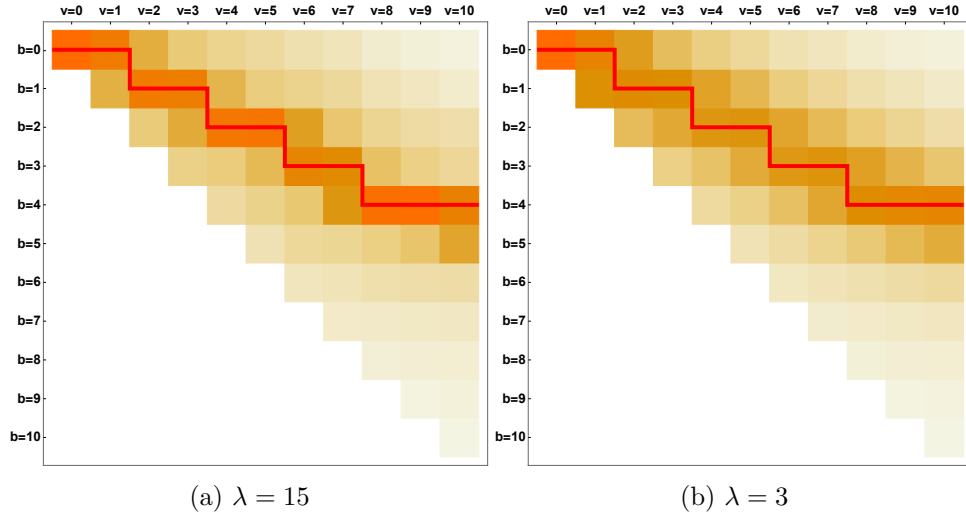


Figure 1: Numerical QRE of the standard first-price auction for two different values of  $\lambda$ . The rows represent the probability a certain offer is submitted for each of the types (rows). A darker shade represents a higher probability. The red line marks the offer with the highest probability for each type.

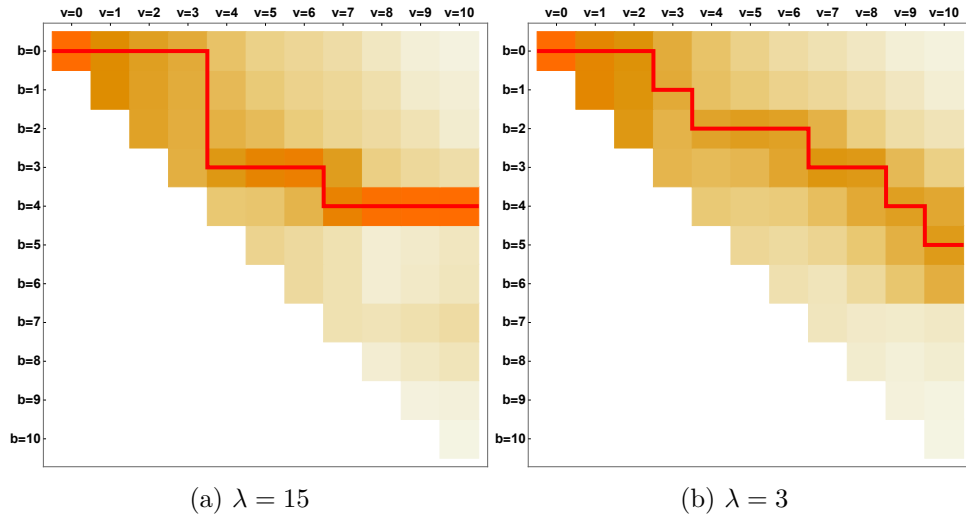


Figure 2: Numerical QRE of the first-price auction with renegotiation **for the bidders** for two different values of  $\lambda$ . The rows represent the probability a certain offer is submitted for each of the types (rows). A darker shade represents a higher probability. The red line marks the offer with the highest probability for each type.

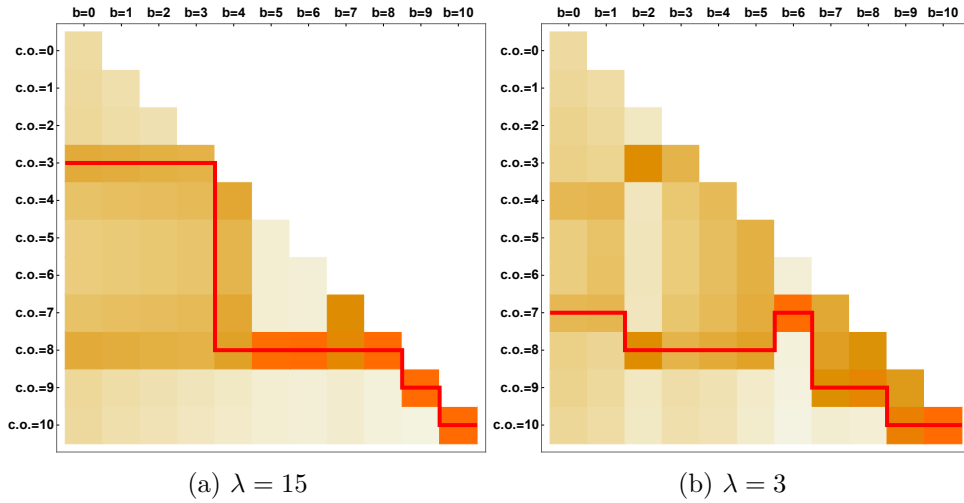


Figure 3: Numerical QRE of the first-price auction with renegotiation **for the auctioneer** for two different values of  $\lambda$ . The rows represent the probability a certain counteroffer is submitted after a certain highest offer (columns) was observed. A darker shade represents a higher probability. The red line marks the counteroffer with the highest probability for each highest offer.

renegotiation and feedback (FPRF). In all settings, the valuations of the bidders are drawn from the set  $\{0, 1, \dots, 100\}$  ECU, all valuations are equally likely. In the FPA treatment, both bidders can submit offers  $\in \{0, 1, \dots, 100\}$  in a first stage. The auctioneer then observes these offers and chooses one of them at will. In the FPR treatment, the auctioneer can additionally make a counteroffer. The counteroffer is automatically accepted if it is below or equal to the value of the chosen bidders, and is rejected if it is higher than his value. This is done to reduce noise from an additional decision of the participants.

The FPRF treatment includes additional feedback for the auctioneer: After each round finishes, the offers and values of the two bidders are revealed to her. With standard preferences, this does not have any implications on the equilibrium bidding strategies derived in 2.1.

### 3.2 Organization

The experiments were conducted in the Cologne Laboratory for Economic Research (CLER) at the University of Cologne, Germany. Using the recruiting system

ORSEE (Greiner, 2015), we invited a random sample of the CLER’s subject pool via email with cash as the only incentive offered. Our participants were mostly students at the University of Cologne, mostly undergraduates, from a variety of majors, and they therefore represent the larger university community. The whole experiment was computerized using the programming environment oTree (Chen, Schonger, and Wickens, 2016). Upon their arrival at the laboratory, participants were seated in visually isolated cubicles and read instructions on their screens (see Appendix 5.1) describing the rules of the game. Following this, they were handed control questions which they had to answer correctly to proceed.

In total, 138 subjects participated in the experiment, with 36 subjects participating in the FPA treatment, 48 subjects participating in the FPR treatment and 54 subjects participating in the FPRF treatment.

Payoffs were stated in ECU, the conversation rate used was  $1\text{ECU} = 0.01\text{ EUR}$ . Participants were paid out in private after the completion of the experiment. All 138 participants were paid their total net earnings. The average payoff for the entire experiment was 16.17 EUR corresponding to approx. 18.95 USD at the time of the payment.

Participants were randomly assigned to one of two rooms where two different treatments were conducted simultaneously. We randomly assigned one of the two roles, bidder and auctioneer, to every participant. Participants kept their assigned role for the whole experiment. Participants were grouped into cohorts of six where two auctioneers and four bidders were matched randomly in each of the 50 rounds within a cohort.

### 3.3 Hypothesis

Our theory predicts that offers in the FPR and FPRF treatments are not correlated with the value of the respective bidder, they submit offers of zero in equilibrium. With this, we can state the following hypotheses:

**Hypothesis 1.** *There is no correlation between the value and the offers in the FPR and FPRF treatments.*



**Hypothesis 2.** *Offers are lower in the FPR and FPRF treatments than in the FPA treatment.*

When we compare offers between the FPR and the FPRF treatment, the difference in feedback could improve learning in the FPRF treatment. The equilibrium bidding strategy in this setting requires a certain depth of reasoning, a bidder needs to understand that any separating bidding strategy leads to full surplus extraction. The additional feedback allows the auctioneer to see how much money she “left on the table” in each round. This, in turn, should lead to higher counteroffers which should lead bidders to adjust their offers downwards. Thus, we expect that the additional feedback pushes bidders closer to the equilibrium bidding strategy. This is also related to our QRE results from subsection 2.2: The additional feedback could lead to less errors, or a higher  $\lambda$  value.

**Hypothesis 3.** *Offers in the FPRF treatment are lower than in the FPR treatment.*

Theory predicts that counteroffers of the auctioneer do not depend on the offers received in the first stage.

**Hypothesis 4.** *There is no correlation between the offer of the chosen bidder and the counteroffer of the auctioneer in the FPR and FPRF treatments.*

The next hypothesis concerns the revenue of the auctioneer. In the FPA treatment, the competition between bidders helps the auctioneer while in the FPR and FPRF treatments, she can only propose the ex-ante optimal take-it-or-leave-it offer to one of the bidders. A numerical simulation confirms this intuition. We can also approximate the revenues with continuous types, since our bid grid is fine enough. For uniformly distributed values,  $F = \mathcal{U}[0, 100]$ , the bidding strategy simplifies to

$$\beta^I(v) = \frac{v}{2}. \quad (15)$$

The expected revenue for the first-price auction for the uniform distribution over the interval  $[0, 100]$  is given by

$$\mathbb{E}[R] = \frac{100}{3}. \quad (16)$$

The optimal take-it-or-leave-it offer in the same setting is given by 50. Offering this to one of the bidders at random results in an expected revenue of  $\frac{100}{4} = 25$ .

**Hypothesis 5.** *The auctioneer's revenue is strictly higher in the FPA treatment than in the FPR and FPRF treatments.*

If the additional feedback of the FPRF really leads to less errors and with that to a QRE that is closer to the unique Bayes-Nash equilibrium, than revenue should be lower in the FPRF treatment.

**Hypothesis 6.** *The auctioneer's revenue is lower in the FPRF treatment than in the FPR treatment.*

Related to Proposition 3, the FPA should be efficient, while theory predicts that the FPR and the FPRF are not.

**Hypothesis 7.** *The FPA is more efficient, in the sense that the bidder with the highest value is more often the winner, than in the FPR and FPRF treatments.*

For the FPA treatment, we should observe that bidders bid according to the equilibrium bidding strategy (15).

**Hypothesis 8.** *Bidders submit offers according to the equilibrium bidding function of  $\beta^I(v) = v/2$  in the FPA treatment.*

### 3.4 Results

We begin with the hypotheses concerning the bidding strategy of the bidders in FPR and FPRF treatments and the comparison with the FPA treatment, Hypothesis 1 and Hypothesis 2.

As a reminder, the equilibrium offers are given by zero in these two settings. However, we observe only four out of 68 bidders who submit an offer of zero when their value is larger than five and of these, only three do so more than once. Also as can be seen in table 2, value has a significant influence on the offers in the FPR and FPRF treatments. Thus, we must reject Hypothesis 1.

In the FPR treatment, the average offer is given by 36.82, while in the FPRF treatment, it is given by 34.25, see table 1. In the FPA treatment, the average

	Mean	Std. Dev.	Min	Max
<b>FPA</b>				
Participants	36	–	–	–
Values	50.56	29.41	0	100
Offers	34.56	21.46	0	95
<b>FPR</b>				
Participants	48	–	–	–
Values	49.58	28.97	0	100
Offers	36.81	23.19	0	98
Counteroffers	53.08	18.24	1	100
<b>FPRF</b>				
Participants	54	–	–	–
Values	50.52	28.99	0	100
Offers	34.25	21.73	0	99
Counteroffers	53.49	18.66	1	99

Table 1: Summary statistics for the treatments.

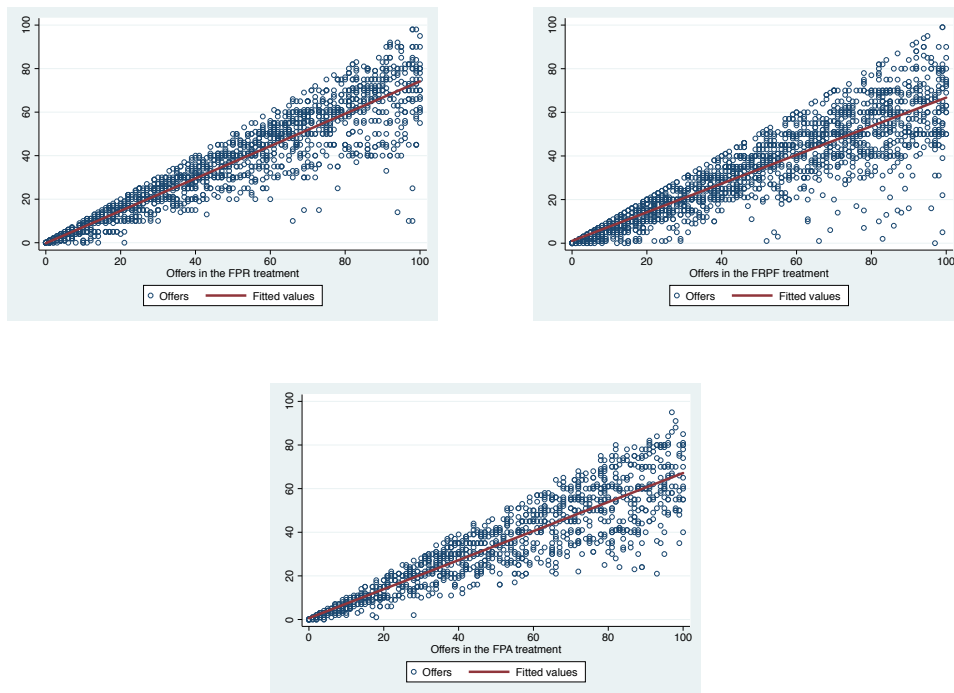


Figure 4: Offers and the corresponding linear regressions in the FPR (left), the FPRF (right) and the FPA treatment (below).

offer is given by 34.56. While the treatment dummy for the FPR treatment has a significant effect on the offers (see Table 3), it is positive, contrary to Hypothesis 2. We find no significant difference in the offers between the FPRF and the FPA treatments. Thus, we must reject Hypothesis 2 as well.

**Result to Hypothesis 1** Offers are correlated with the respective values in the FPR and FPRF treatments ( $p=0.000$ , linear regression).

**Result to Hypothesis 2** There is no significant difference between the offers in the FPA and the FPRF treatment (linear regression,  $p = 0.6897$ ). Offers are significantly higher in the FPR treatment than in the FPA treatment (linear regression,  $p = 0.097$ ).

However, offers in the FPR are significantly higher than in the FPRF treatment (students' t-Test  $p = 0.06$ ). This can also be seen in Table 3.

**Result to Hypothesis 3** Offers in the FPRF treatment are significantly lower than in the FPR treatment (students' t-Test  $p = 0.0587$ ).

The average counteroffer is given by 53.08 in the FPR treatment and 53.49 in the FPRF treatment, which are both slightly higher than the ex-ante optimal take-it-or-leave-it offer of 50 in the unique equilibrium. A regression of the counteroffer on the offer of the chosen bidder suggests a high correlation between the two in both treatments (see Table 4). Therefore we must reject Hypothesis 4, as predicted by our analysis of the QRE (Proposition 6) in the FPR setting.

**Result to Hypothesis 4** Counteroffers in the FPR and FPRF treatments are correlated with the offer of the chosen bidder. (linear regression  $p=0.000$ ).

The revenues for the auctioneer are very similar in all three treatments (means: FPA: 46.47 FPR: 47.96 FPRF: 45.37). All three average revenues are higher than expected from theory but with a prediction of around 33 ECU in the first-price auction setting and around 25 ECU in the FPR and FPRF treatments (numerical

simulations), we can conclude that the auctioneers were able to exploit some of the private information shared by the bidders.

**Result to Hypothesis 5** There is no significant difference between the revenues in the FPR and FPRF treatments with respect to the FPA treatment (students' t-Test: FPA-FPR:  $p = 0.6883$ ; FPA-FPRF:  $p = 0.3652$ ).

The difference between the FPR and the FPRF treatment is indeed significant.

**Result to Hypothesis 6** The revenue in the FPRF treatment is significantly lower than in the FPR treatment (students' t-Test:  $p = 0.0757$ ).

Regarding the efficiency, we observe no significant differences between the treatments.

**Result to Hypothesis 7** There is no significant difference concerning the efficiency between the FPA and the FPR, and the FPA and the FPRF (students' t-Test, FPA-FPR:  $p = 0.6310$ ; FPA-FPRF:  $p = 0.2929$ ).

Summary statistics for the FPA treatment can be found in table 1. We observe overbidding in line with the experimental literature, the average offer is given by 34.56, the median offer is 33. From table 2, we must reject *Hypothesis 5*. The slope is significantly different from 0.5.

**Result to Hypothesis 8** Bidders bid significantly higher than predicted in the FPA treatment (students' t-Test  $p = 0.000$ ).

## 4 Conclusion

In this paper, we investigate how bidders react to commitment in first-price auctions in a simple and concise setting. While theory clearly predicts that the offers of the bidders should be higher in the standard first-price auction than in the first-price auction with renegotiation, we cannot verify this experimentally. The same holds true for the revenue of the auctioneer and the efficiency of the mechanisms, however, we find evidence for neither hypothesis. Offers are informative of the bidders' type

Dependent variable: Offer						
Treatment:	FPA		FPR		FPRF	
Value	0.66*** (22.35)	0.66*** (22.33)	0.74*** (31.16)	0.74*** (31.14)	0.66*** (28.78)	0.66*** (28.88)
$\beta_0$	0.85* (1.66)	1.20 (1.16)	-0.24 (-0.44)	0.30 (0.26)	0.88 (1.47)	1.92** (2.00)
Period		-0.01 (-0.36)		-0.02 (-0.56)		-0.04 (-1.46)
Observations	1200		1600		1800	

*t* statistics in parentheses

\*  $p < 0.1$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.001$

Table 2: Panel regression estimates for the offers in the FPR, FPRF and FPA treatments

in the first-price auction with renegotiation but the auctioneers are not able to lever this information into profit. This could reduce the pressure to pool of the bidders, as the quantal response equilibrium analysis insinuates. For real-life procurement, this would mean that a buyer does not need to focus on the commitment of her mechanism and can expect competitive offers, even when the rules on how a winner is selected are not clear. On the other hand, there have been studies that show a strong reaction to a lack of commitment by laboratory participants. This opens the door for further research. For example, it would be interesting to understand how a mechanism can convey commitment in a way that bidders understand and react to varying amounts of it.

	Offer
Value	0.689*** (46.24)
Period	-0.0146 (-0.38)
FPR	3.249* (1.66)
FPRF	0.751 (0.40)
FPR $\times$ Period	-0.00612 (-0.11)
FPRF $\times$ Period	-0.0282 (-0.58)
Constant	-0.232 (-0.17)
Observations	4600

*t* statistics in parentheses

\*  $p < 0.05$ , \*\*  $p < 0.01$ , \*\*\*  $p < 0.001$

Table 3: Panel regression estimates for the effect of the treatment variables on the offers of the bidders

<b>Dependent variable: Counteroffer</b>		
	FPR	FPRF
Offer of chosen bidder	0.798*** (53.39)	0.919*** (72.56)
Period	0.00582 (0.29)	0.0576*** (3.58)
Constant	12.75*** (9.02)	9.089*** (8.45)
Observations	800	900

*t* statistics in parentheses

\*  $p < 0.05$ , \*\*  $p < 0.01$ , \*\*\*  $p < 0.001$

Table 4: Panel regression estimates for the effect of the treatment variables on the counteroffers of the auctioneers

## 5 Appendix

### 5.1 Instructions

#### 5.1.1 FPA Treatment

### Herzlich willkommen zum Experiment

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Herzlichen Dank für Ihre Teilnahme an diesem Experiment. Während des Experiments ist es Ihnen nicht erlaubt, mit anderen Teilnehmern zu kommunizieren, Mobiltelefone zu benutzen, oder andere Programme auf dem Computer zu starten. Sollten Sie gegen diese Regeln verstoßen, müssen wir Sie leider vom Experiment und all seinen Auszahlungen ausschließen. Im Anschluss an die Präsentation erhalten Sie die Anleitung noch einmal in gedruckter Form und müssen Verständnisfragen beantworten. Das eigentliche Experiment startet erst, wenn alle Teilnehmer alle Verständnisfragen korrekt beantwortet haben. Falls Sie Fragen haben, heben Sie bitte die Hand. Ein Experimentleiter wird dann an Ihren Platz kommen, um Ihre Frage leise zu beantworten.

### Übersicht

---

Das Experiment besteht aus 50 Runden, die jeweils aus zwei Stufen bestehen und die gleiche Abfolge an Entscheidungen haben.

In diesem Experiment gibt es zwei Rollen. Ein Verkäufer, der ein Gut verkaufen möchte und Interessenten für das Gut. Zu Beginn des Experiments wird Ihnen zufällig eine der beiden Rollen zugewiesen. Sie behalten diese Rolle über das gesamte Experiment.

In jeder Runde werden neue Gruppen bestehend aus 2 Interessenten und einem Verkäufer zufällig gebildet.

In der ersten Stufe gibt jeder Interessent ein Angebot für das Gut ab.

In der zweiten Stufe beobachtet der Verkäufer die Angebote der Interessenten und wählt eines der Angebote aus.

Ihr Gewinn aus dem Experiment entspricht der Summe Ihrer Gewinne aus allen Runden, dabei entspricht 1 ECU einem Cent.

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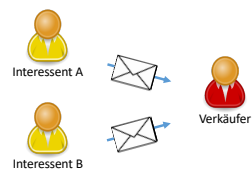
Stufe 1	Stufe 2
<p>Interessant A Interessant B Verkäufer</p>	<p>Verkäufer Interessant A <b>ODER</b> Verkäufer Interessant B</p>

Figure 5: Instructions page 1 and 2 for the FPA treatment



## Stufe 1: Angebotsabgabe der Interessenten

1. Zu Beginn der ersten Stufe beobachtet jeder Interessent, welchen Wert das Gut für ihn hat, dabei ist jede ganze Zahl zwischen 0 ECU und 100 ECU gleich wahrscheinlich.
2. Die Interessenten geben jeweils ein ganzzahliges Angebot an den Verkäufer ab. Dieses Angebot muss kleiner als oder gleich dem Wert des Gutes für den Interessenten sein.



## Stufe 2: Angebotsannahme des Verkäufers

1. Der Verkäufer beobachtet die beiden Angebote der Interessenten.
2. Der Verkäufer wählt eines der beiden Angebote aus.

### Gewinn der Teilnehmer:

- Der Gewinn des ausgewählten Interessenten:  $\text{Wert} - \text{Angebot} + 5 \text{ ECU}$ .
- Der Gewinn des nicht ausgewählten Interessenten:  $5 \text{ ECU}$ .
- Der Gewinn des Verkäufers:  $\text{Angebot}$ .



Figure 6: Instructions pages 3 and 4 for the FPA treatment

## 5.1.2 FPR Treatment

### Herzlich willkommen zum Experiment

Herzlichen Dank für Ihre Teilnahme an diesem Experiment. Während des Experiments ist es Ihnen nicht erlaubt, mit anderen Teilnehmern zu kommunizieren, Mobiltelefone zu benutzen, oder andere Programme auf dem Computer zu starten. Sollten Sie gegen diese Regeln verstoßen, müssen wir Sie leider vom Experiment und all seinen Auszahlungen ausschließen. Im Anschluss an die Präsentation erhalten Sie die Anleitung noch einmal in gedruckter Form und müssen Verständnisfragen beantworten. Das eigentliche Experiment startet erst, wenn alle Teilnehmer alle Verständnisfragen korrekt beantwortet haben. Falls Sie Fragen haben, heben Sie bitte die Hand. Ein Experimentleiter wird dann an Ihren Platz kommen, um Ihre Frage leise zu beantworten.

### Übersicht

Das Experiment besteht aus 50 Runden, die jeweils aus zwei Stufen bestehen und die gleiche Abfolge an Entscheidungen haben.

In diesem Experiment gibt es zwei Rollen. Ein Verkäufer, der ein Gut verkaufen möchte und Interessenten für das Gut. Zu Beginn des Experiments wird Ihnen zufällig eine der beiden Rollen zugewiesen. Sie behalten diese Rolle über das gesamte Experiment.

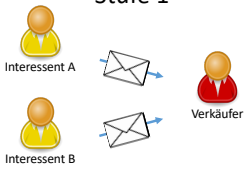
In jeder Runde werden neue Gruppen bestehend aus 2 Interessenten und einem Verkäufer zufällig gebildet.

In der ersten Stufe gibt jeder Interessent ein Angebot für das Gut ab.

In der zweiten Stufe beobachtet der Verkäufer die Angebote der Interessenten und wählt einen der Interessenten aus, um diesem ein Gegenangebot zu machen.

Ihr Gewinn aus dem Experiment entspricht der Summe Ihrer Gewinne aus allen Runden, dabei entspricht 1 ECU einem Cent.

#### Stufe 1

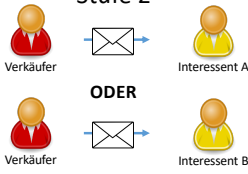


Interessant A

Interessant B

Verkäufer

#### Stufe 2



Verkäufer

Interessant A

**ODER**

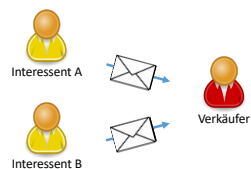
Verkäufer

Interessant B

Figure 7: Instructions page 1 and 2 for the FPR treatment

## Stufe 1: Angebotsabgabe der Interessenten

1. Zu Beginn der ersten Stufe beobachtet jeder Interessent, welchen Wert das Gut für ihn hat, dabei ist jede ganze Zahl zwischen 0 ECU und 100 ECU gleich wahrscheinlich.
2. Die Interessenten geben jeweils ein ganzzahliges Angebot an den Verkäufer ab. Dieses Angebot muss kleiner als oder gleich dem Wert des Gutes für den Interessenten sein.



## Stufe 2: Gegenangebot des Verkäufers

1. Der Verkäufer beobachtet die beiden Angebote der Interessenten.
2. Der Verkäufer wählt einen der beiden Interessenten aus und macht diesem ein ganzzahliges Gegenangebot.
3. Das Gegenangebot des Verkäufers wird automatisch vom ausgewählten Interessenten akzeptiert, wenn es unter dem Wert des Gutes für den Interessenten liegt. Es wird automatisch vom ausgewählten Interessenten abgelehnt, wenn es über dem Wert des Gutes für den Interessenten liegt.

### Gewinn der Teilnehmer:

- a) Wird das Gegenangebot akzeptiert, ...
  - ... ist der Gewinn des ausgewählten Interessenten:  $\text{Wert} - \text{Gegenangebot} + 5 \text{ ECU}$ .
  - ... ist der Gewinn des nicht ausgewählten Interessenten:  $5 \text{ ECU}$ .
  - ... ist der Gewinn des Verkäufers: **Gegenangebot**.
- b) Wird das Gegenangebot abgelehnt, ...
  - ... ist der Gewinn des ausgewählten Interessenten:  $5 \text{ ECU}$ .
  - ... ist der Gewinn des nicht ausgewählten Interessenten:  $5 \text{ ECU}$ .
  - ... ist der Gewinn des Verkäufers:  $0 \text{ ECU}$ .



Figure 8: Instructions pages 3 and 4 for the FPR treatment

### 5.1.3 FPRF Treatment

## Herzlich willkommen zum Experiment

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Herzlichen Dank für Ihre Teilnahme an diesem Experiment. Während des Experiments ist es Ihnen nicht erlaubt, mit anderen Teilnehmern zu kommunizieren, Mobiltelefone zu benutzen, oder andere Programme auf dem Computer zu starten. Sollten Sie gegen diese Regeln verstoßen, müssen wir Sie leider vom Experiment und all seinen Auszahlungen ausschließen. Für Ihr Erscheinen zu diesem Experiment erhalten Sie 4€.

Im Anschluss an die Präsentation erhalten Sie die Anleitung noch einmal in gedruckter Form und müssen Verständnisfragen beantworten. Das eigentliche Experiment startet erst, wenn alle Teilnehmer alle Verständnisfragen korrekt beantwortet haben.

Ihre Auszahlung ergibt sich als Summe Ihrer Verdienste aus den beiden Teilen des Experiments und den 4€ für Ihr Erscheinen.

Falls Sie Fragen haben, heben Sie bitte die Hand. Ein Experimentleiter wird dann an Ihren Platz kommen, um Ihre Frage leise zu beantworten.

## Übersicht

---

Das Experiment besteht aus 50 Runden, die jeweils aus zwei Stufen bestehen und die gleiche Abfolge an Entscheidungen haben.

In diesem Experiment gibt es zwei Rollen. Ein Verkäufer, der ein Gut verkaufen möchte und Interessenten für das Gut. Zu Beginn des Experiments wird Ihnen zufällig eine der beiden Rollen zugewiesen. Sie behalten diese Rolle über das gesamte Experiment.

In jeder Runde werden neue Gruppen bestehend aus **zwei Interessenten** und **einem Verkäufer** zufällig gebildet. In der ersten Stufe gibt jeder Interessent ein Angebot für das Gut ab. In der zweiten Stufe beobachtet der Verkäufer die Angebote der Interessenten und wählt einen der Interessenten aus, um diesem ein Gegenangebot zu machen.

Ihr Gewinn aus dem Experiment entspricht der Summe Ihrer Gewinne aus allen Runden plus 4€ für Ihr Erscheinen, dabei entspricht 1 ECU einem Cent.

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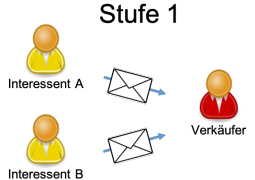
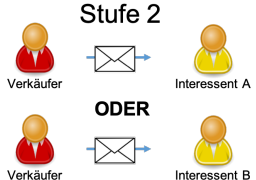
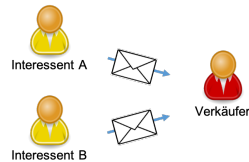
<h3>Stufe 1</h3> 	<h3>Stufe 2</h3> 
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Figure 9: Instructions page 1 and 2 for the FPRF treatment

## Stufe 1: Angebotsabgabe der Interessenten

1. Zu Beginn der ersten Stufe beobachtet jeder Interessent, welchen Wert das Gut für ihn hat, dabei ist jede ganze Zahl zwischen 0 ECU und 100 ECU gleich wahrscheinlich.
2. Die Interessenten geben jeweils ein ganzzahliges Angebot an den Verkäufer ab. Dieses Angebot muss kleiner als oder gleich dem Wert des Gutes für den Interessenten sein.



## Stufe 2: Gegenangebot des Verkäufers

1. Der Verkäufer beobachtet die beiden Angebote der Interessenten.
2. Der Verkäufer wählt einen der beiden Interessenten aus und macht diesem ein ganzzahliges Gegenangebot.
3. Das Gegenangebot des Verkäufers wird automatisch vom ausgewählten Interessenten akzeptiert, wenn es unter dem Wert des Gutes für den Interessenten liegt. Es wird automatisch vom ausgewählten Interessenten abgelehnt, wenn es über dem Wert des Gutes für den Interessenten liegt.
4. Erst **nachdem** der Verkäufer sein Gegenangebot verbindlich abgegeben hat, kann er sehen welchen Wert das Gut für den jeweiligen Interessenten in dieser Runde hatte.

**Gewinn der Teilnehmer in dieser Runde:**

a) Wird das Gegenangebot akzeptiert, ...  
... ist der Gewinn des ausgewählten Interessenten:  $\text{Wert} - \text{Gegenangebot} + 5 \text{ ECU}$ .  
... ist der Gewinn des nicht ausgewählten Interessenten: **5 ECU**.  
... ist der Gewinn des Verkäufers: **Gegenangebot**.

b) Wird das Gegenangebot abgelehnt, ...  
... ist der Gewinn des ausgewählten Interessenten: **5 ECU**.  
... ist der Gewinn des nicht ausgewählten Interessenten: **5 ECU**.  
... ist der Gewinn des Verkäufers: **0 ECU**.



Figure 10: Instructions pages 3 and 4 for the FPRF treatment

## 5.2 Logit QRE for the FPA

The action space of the bidders is given by  $A^B = T = \{t_i\}_{i \in \{0, \dots, 10\}}$ . The auctioneer's action space is given by  $A^A = \{b^1, b^2\} = \{a_i^A\}_{i \in \{1, \dots, 2\}}$ , either she chooses the offer of bidder one or the offer of bidder two. The expected utility of bidder 1 being of type  $t_i$  and submitting an offer of  $t_j$  is given by

$$U_1^B(t_i, t_j, \sigma^B, \sigma^A) = \underbrace{\sum_{k=0}^{10} \sum_{l=0}^k \sigma_{kl}^B}_{\text{bidder 2: } b^2 \text{ action.: } a_m^A} \sum_{m=1}^2 \sigma_{jlm}^A \begin{cases} t_i - t_j & a_m^A = b^1 \\ 0 & a_m^A \neq b^1 \end{cases} \quad (17)$$

$$= \sum_{k=0}^{10} \sum_{l=0}^k \sigma_{kl}^B \sigma_{j1l}^A (t_i - t_j) \quad (18)$$

$$:= U_{1,ij}^B(\sigma^B, \sigma^A). \quad (19)$$

Analogously, the expected utility of bidder 2 being of type  $t_i$  and submitting an offer of  $t_j$  is given by

$$U_2^B(t_i, t_j, \sigma^B, \sigma^A) = \sum_{k=0}^{10} \sum_{l=0}^k \sigma_{kl}^B \sum_{m=1}^2 \sigma_{jlm}^A \begin{cases} t_i - t_j & a_m^A = b^2 \\ 0 & a_m^A \neq b^2 \end{cases} \quad (20)$$

$$= \sum_{k=0}^{10} \sum_{l=0}^k \sigma_{kl}^B \sigma_{j2l}^A (t_i - t_j) \quad (21)$$

$$:= U_{2,ij}^B(\sigma^B, \sigma^A). \quad (22)$$

The expected utility of the auctioneer having received the offers  $b^1 = t_i$  and  $b^2 = t_j$  and taking action  $a_k^A$  is given by

$$U_{ijk}^A(\sigma^B, \sigma^A) = \begin{cases} t_i & a_k = b^1 \\ t_j & a_k = b^2 \end{cases} \quad (23)$$

This yields the following system of equations,

$$\begin{aligned}
\sigma_{ij}^B &= \frac{e^{\lambda U_{1,ij}^B(\sigma^B, \sigma^A)}}{\sum_{k=0}^i e^{\lambda U_{1,ik}^B(\sigma^B, \sigma^A)}} \quad \forall t_i, t_j \in T \\
\sigma_{ij}^B &= \frac{e^{\lambda U_{2,ij}^B(\sigma^B, \sigma^A)}}{\sum_{k=0}^i e^{\lambda U_{2,ik}^B(\sigma^B, \sigma^A)}} \quad \forall t_i, t_j \in T \\
\sigma_{ijk}^A &= \frac{e^{\lambda U_{ijk}^A(\sigma^B, \sigma^A)}}{\sum_{m=1}^2 e^{\lambda U_{ijm}^A(\sigma^B, \sigma^A)}} \quad \forall t_i, t_j \in T \text{ and } \forall a_k^A \in A^A.
\end{aligned} \tag{24}$$

## References

- CAI, G. G., P. R. WURMAN, AND X. GONG (2010): “A Note on Discrete Bid First-Price Auction With General Value Distribution,” *International Game Theory Review*, 12(01), 75–81.
- CHE, Y.-K. (1993): “Design competition through multidimensional auctions,” *The RAND Journal of Economics*, pp. 668–680.
- CHEN, D. L., M. SCHONGER, AND C. WICKENS (2016): “oTree—An open-source platform for laboratory, online, and field experiments,” *Journal of Behavioral and Experimental Finance*, 9, 88 – 97.
- CHWE, M. S.-Y. (1989): “The discrete bid first auction,” *Economics Letters*, 31(4), 303–306.
- FUGGER, N., E. KATOK, AND A. WAMBACH (2016): “Collusion in Dynamic Buyer-Determined Reverse Auctions,” *Management Science*, 62(2), 518–533.
- GOEREE, J. K., C. A. HOLT, AND T. R. PALFREY (2016): *Quantal Response Equilibrium - A Stochastic Theory of Games*. Princeton University Press, Kassel.
- GREINER, B. (2015): “Subject pool recruitment procedures: organizing experiments with ORSEE,” *Journal of the Economic Science Association*, 1(1), 114–125.
- JAP, S. D. (2002): “Online reverse auctions: Issues, themes, and prospects for the future,” *Journal of the Academy of Marketing Science*, 30(4), 506–525.
- KRISHNA, V. (2009): *Auction theory*. Academic press.
- MCAFEE, R., AND D. VINCENT (1997): “Sequentially Optimal Auctions,” *Games and Economic Behavior*, 18(2), 246 – 276.
- MCKELVEY, R. D., AND T. R. PALFREY (1995): “Quantal response equilibria for normal form games,” *Games and economic behavior*, 10(1), 6–38.



- TAN, G. (1996): “Optimal Procurement Mechanisms for an Informed Buyer,” *The Canadian Journal of Economics / Revue canadienne d’Economie*, 29(3), 699–716.
- VARTIAINEN, H. (2013): “Auction design without commitment,” *Journal of the European Economic Association*, 11(2), 316–342.



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