Discussion Paper No. 17-022

Evaluating Regulation within an Artificial Financial System – A Framework and its Application to the Liquidity Coverage Ratio Regulation

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Abstract

We develop a general model of the financial system that allows for the evaluation of bank regulation. Our framework comprises the agents and institutions that have proved crucial in the propagation of the subprime mortgage shock in the U.S. into a global financial crisis: Commercial banks and investment banks, which can also be interpreted as shadow banks, interact on wholesale debt markets. Beside a market for short term interbank loans and long term bank bonds, other funding sources include insured customer deposits, uninsured investor deposits and repos. While credit to the real sector is the principal asset of commercial banks, investment banks specialize in trading securities, which may differ according to risk, maturity and liquidity. As a first application of the model we implement the liquidity coverage ratio (LCR) regulation and analyze its impact on bank balance sheets, interest rates, the transmission of monetary policy and the stability of bank lending in the face of shocks. We find that the LCR regulation reduces the supply of loans to the real sector, increases the maturity and interest rate of long term wholesale debt, and strongly diminishes the role of the overnight interbank market as a funding source. Our simulations suggest that the transmission of changes to short term monetary policy rates is severely impaired when the LCR regulation is binding. Furthermore, we find that a strong confidence shock can lead to a protracted credit crunch under the liquidity regulation.

Keywords: financial system - agent-based model - liquidity coverage ratio

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1 Introduction

Starting in February 2007, increasing defaults in subprime mortgages in the U.S. shocked the financial sector. By September 2008, the relatively small and local losses from subprime mortgages had developed into a global financial crisis, which eventually threw many economies around the world into recession. The channels through which the initial losses propagated were manifold, but they were by no means unknown. The theory of fire sales introduced by Shleifer and Vishny (1992), the financial accelerator model of Bernanke and Gertler (1989), the theory of bank runs developed by Diamond and Dybvig (1983) and the financial contagion models introduced by Allen and Gale (2000) arguably constitute a sufficient intellectual basis for understanding the individual mechanisms that played out during the financial crisis. All of these contributions were acknowledged as being seminal well before the dawn of the crisis. Hence, there was no lack of basic understanding of mechanisms but of in-depth knowledge of the institutional structure of the financial system, which includes the interdependencies and the interactions of the relevant business models and individual agents. Over the past decade, a plethora of theoretical and empirical studies has substantially advanced this knowledge on many fronts. However, a holistic view of the financial system remains a challenge. This is not least due to the complexity that ensues from the interplay of heterogeneous institutions such as traditional retail banks, wholesale funded banks, investment banks, hedge funds, money market funds, insurance companies and pension funds. Institutions' balance sheets are interconnected to form a dynamic network that responds, for example, to financial stress, phase changes in the business cycle, monetary policy and new regulation. It is thereby not only the structure of the network that is affected. Shifts in agents' behavior could preserve the individual links of the network but change the way shocks propagate through the network. For example, it is of great significance for the stability of the system whether lender-borrower relationships are formed through short-term or long-term loans, or whether the assets that overlap between agents' portfolios are more or less liquid. In this regard, we believe that the way forward towards a better understanding of the financial system as a whole will require a stronger focus on interactions in the financial sector. Taking the interactions of agents into account will facilitate higher precision in the design and impact assessment of new regulation. Furthermore, unintended side effects of regulation, which are often the consequence of a system's complexity, can be better anticipated.

In this paper, we develop an extensive agent-based model of the financial sector. The agent-based approach is particularly well suited with regard to facilitating a holistic, but detailed view on the financial system. By modeling the decentralized decision making process of heterogeneous banks and by letting them interact on asset and funding markets, we allow for complex dynamics to emerge. Thereby we avoid issues of mathematical tractability, which would pose limitations to the scope of the model in a more standard partial equilibrium setting. Somewhat atypically, the model is not designed with a specific question in mind, but rather as a general framework that can be quickly adapted to address a variety of questions concerning, for example, financial stability, expectation formation, asset pricing, monetary policy transmissions etc. In particular, we will use the framework as a laboratory for policy evaluations. Thereby, the extensive scope of the model allows us to implement existing regulation in unprecedented detail and to assess its likely impact on several decision areas.

Our paper contributes to the existing literature in two domains. The first contribution is of technical nature, i.e. we propose ways of modeling important features of the financial system. The concepts we introduce allow for a rich bank balance sheet structure that includes assets of differing maturity and liquidity as well as diverse debt forms such as short and long term secured and unsecured funding instruments. We endow the agents of the model, which represent either a commercial bank or an investment bank with sophisticated tools for managing their balance sheets. The portfolio and the capital structure choices are explicitly optimized and take into account risk, return/cost and maturity. Interest rates and prices are endogenously computed with a novel mechanism. The second contribution of the paper pertains to the application of the model. We implement the liquidity coverage ratio (LCR) regulation in unprecedented detail and assess its impact on balance sheets, interest rates, monetary policy and some aspects of financial stability.[1] The results we obtain from simulations confirm existing impact assessments but also suggest novel impact channels. Specifically, we find that the LCR regulation will reduce loans to the real sector, increase long term interest rates and drastically diminish the role of the overnight interbank market as a source of funding. Concerning its impact on monetary policy, our simulations suggest two opposing effects: While the transmission of changes to short term policy rates is severely impaired when the LCR regulation is binding, the transmission of changes to the volume of customer deposits as a consequence of monetary policy is likely to have a slightly more pronounced effect on bank lending. In our analysis of financial stability, we focus on commercial banks' loan supply to the real sector. Emulating the loss of confidence banks experienced after the collapse of Lehman Brothers in September 2008, we find that the LCR regulation is unlikely to stabilize the loan supply. On the contrary, when the LCR regulation is binding, a strong confidence shock can lead to a protracted credit crunch by destabilizing the creditors of commercial banks. A solvency shock, on the other hand, is initially slightly less detrimental when the LCR regulation is binding. However, generally lower profit rates for commercial banks under the LCR regulation may lead to a slower recovery from the shock.

The rest of this paper is organized as follows: We proceed in Section 2 with a brief review of the related literature. Section 3 introduces our model and derives the behavior of each agent type in detail. In Section 4, we extend the model by the liquidity coverage ratio regulation. Simulation results are presented in Section 5 and Section 6 concludes.

2 Related Literature

A general model of the financial system naturally relates to a large body of literature. First, the empirical literature that sketches developments before and during the financial crisis serves as a rough guide to our modeling choices with regard to the relevant agents and institutions. We thereby assume that the financial crisis has revealed the relevant transmission channels of financial stress that we intend to include in our model. Controlling these transmission channels has also been the principal aim of the regulation that has been introduced as a response to the crisis and whose assessment we want to facilitate with our model. More specifically, our choice to include two types of bank business models is motivated by the fact that large broker dealer banks (and investment banks) were at the core of the financial crisis. Their highly overlapping portfolios[2], central position in the interbank network[3] and prevailing short term funding sources[4]

[1]An impact assessment of the LCR regulation is chosen as the first application of our framework because it potentially affects all parts of banks' balance sheets (asset composition, asset liquidity, funding structure and maturity structure). Therefore, it is well suited to demonstrate the workings of our model and provides a good overview of the types of analysis that can be conducted.

[2]Blei and Ergashev (2014) e.g. measure a buildup of systemic risk in the US due to asset commonality in the run up of the financial crisis. The higher the overlap, the greater the spill-overs will be as a consequence of fire sales. Theoretical and empirical studies have studied the existence and implications of fire sales in financial markets (see e.g. Brunnermeier and Pedersen (2009) and Merrill et al. (2012), respectively).

[3]Craig and Von Peter (2014) e.g. find that being a large primary dealer or international bank in the German interbank network raises the probability of being at the core of the interbank market.

[4]Adrian and Shin (2010) document that the short term funding, typically in the form of repos, induces a procyclical relationship between leverage growth and asset growth for large investment banks. Gorton and Metrick (2012) argue that a run on repos was at the heart of the financial crisis.

rendered them a catalyst for financial stress. Furthermore, the strong link between investment banks and commercial banks through wholesale funding boosted spillovers into the real sector.[5]

Our paper is related to the strands of literature that study the theory of financial shock propagation. Shocks can propagate directly through creditor-debtor relationships on the interbank market or, in combination with fire sales, indirectly through overlapping portfolios.[6] In particular, the literature on interbank networks has flourished in the wake of the financial crisis. Typically an interbank network is constructed on the computer in order to analyze the transmission of an exogenous shock to one of the nodes in the network. The general network structure, the size of shocks, the location of the shocked node in the network and the capital structure of the banks linked within the interbank market have been found to be crucial for the susceptibility of the banking system to systemic risk.[7] However, most network models in the literature comprise of a static structure in which banks do not actively manage their balance sheets. Our paper, on the other hand, is more related to a small but burgeoning literature which model banks as agents that can in one form or the other react to changing circumstances. The behavior of agents thereby changes the dynamics of shock propagation. One of the first models in this spirit is developed by Bluhm and Krahnen (2011). They study a system of three financial institutions that are connected through direct interbank linkages and indirect linkages due to overlapping portfolios. Prices are computed endogenously, which can lead to fire sale dynamics, as institutions, which need to fulfill capital requirements, adjust their portfolios in response to a shock. In Georg (2013), banks optimize their portfolio consisting of a risky asset and riskless excess reserves. They fund their asset side with equity, deposits, interbank loans and central bank loans. The volume of deposits and the return on the risky asset fluctuate randomly, which triggers reactions from banks. The framework is employed in order to compare different network structures with regard to their effect on stability. According to Georg (2013), contagion tends to be less pronounced in scale-free interbank networks than in random and small-world networks. In the agent-based model developed by Fischer and Riedler (2014), agents, which represent financial institutions, optimize a portfolio consisting of a risky asset and cash. The price of the risky asset is determined endogenously through market clearing and thus depends on the expectations of agents. Depending on their past success, agents can follow either a fundamentalist or a chartist strategy when forming expectations. Within this framework, it is shown that when leverage is high and funding short term, overlapping portfolios become a major source of systemic risk. Greenwood et al. (2015) construct a model of fire-sale spillovers that can be estimated with balance sheet data. A bank in their model adjusts its balance sheet according to a specified rule when it is hit by an adverse shock. The adjustment leads to price impacts, which may induce other banks to react. Duarte and Eisenbach (2015) extend this framework in order to build a systemic risk measure that can track vulnerabilities over time. They calibrate their model on U.S. broker-dealers, using data from the tri-party repo market. The calibrated model documents a buildup of systemic risk starting in the early 2000s. Furthermore, it can be inferred that during the financial crisis an exogenous 1% decline in the prices of repo-financed assets would have led to fire sales resulting in a 12% drop in broker-dealers' equity. Halaj and Kok (2015) present an agent-based model, in which the interbank network emerges endogenously from agents' portfo-

[5]Wholesale funding has become an increasingly important funding source for commercial banks since the early 1990s (cf. Feldman and Schmidt, 2001). The consequences of this development were felt during the financial crisis as interbank markets were severely disrupted (see e.g. Afonso et al., 2011). Ivashina and Scharfstein (2010) e.g. find that banks that were more reliant on refinancing their loans to the real sector through wholesale debt rather than deposits displayed a higher decline in lending.

[6]Seminal contributions to the literature on direct linkages include Allen and Gale (2000); Freixas et al. (2000); Eisenberg and Noe (2001). Fire-sale-driven shock propagation is discussed e.g. in Shleifer and Vishny (1992); Kiyotaki and Moore (1997); Brunnermeier and Pedersen (2009).

[7]For a survey of the literature see e.g. Chinazzi and Fagiolo (2013).

lio optimization. Since regulation imposes constraints on the portfolio decisions of agents, the authors can use their model in order to assess the impact of different regulatory measures on the structure of the interbank market and the implied contagion risk. Their findings suggest that while large exposure limits do have a pronounced effect on contagion risk, credit valuation adjustments are less effective. Aldasoro et al. (2015) develop a network model in which banks lend to each other in the interbank market and invest in non-liquid assets. While aggregate positions of interbank assets on balance sheets are the result of portfolio-optimization and market clearing, specific interbank linkages are generated via matching algorithms. When testing the impact of liquidity and capital requirements on their model, they find that although both types of regulation effectively reduce systemic risk, capital requirements result to be less detrimental to overall investment. Montagna and Kok (2016) develop an agent-based model, where agents interact with each other through a multi-layered network model. The linkages of different layers of the model thereby represent interbank relationships of different maturities as well as indirect linkages through portfolio overlap. In their model, banks adjust their balance sheets only when regulatory requirements are violated. The authors find that including the multiple layers of linkages non-linearly amplifies the propagation of shocks.

A last strand of literature that needs mentioning concerns itself with the impact of the liquidity coverage ratio regulation, which is the focus of the simulations in Section 5. The Basel Committee issued a first version of the regulation in December 2010. After its endorsement by the Group of Governors and Heads of Supervision, a revised final version of the LCR was issued in January 2013. Taking into account that the regulation is still in its implementation phase (in the EU it will be fully implemented in 2018), it is no surprise that papers discussing it are scarce and existing impact assessments rather tentative. Nevertheless, some work has been done concerning the regulation's impact on banks' balance sheets, its potential interaction with monetary policy and its effect on financial stability. Cetina and Gleason (2015) provide a review of the literature. When pertinent, we refer to the findings of individual papers in our discussion of the simulation results.

3 The Model

The model, which is schematized in Figure 1, comprises three types of markets: asset markets, funding markets and a banking market. The banking market is populated by heterogeneous agents that can be classified as either commercial bank agents or investment bank agents. Interactions between the two bank business models are confined to wholesale debt markets including an interbank market, which features a stylized core-periphery structure.[8] In our model, and broadly in line with empirical evidence (see e.g. Craig and Von Peter, 2014), a few large investment bank agents at the core provide interbank loans to many small commercial bank agents in the periphery of the banking market. A commercial bank agent follows a rather traditional business model. It takes customer deposits and issues loans to the real sector. Besides using deposits, commercial banks can finance their activities through equity and wholesale debt, which comes in the form of short term overnight loans and long term bank bonds. An exogenous central bank acts as a lender of last resort by providing limitless credit to commercial bank agents at a relative expensive marginal lending rate. Investment bank agents, on the other hand, do not issue loans to the real sector nor do they accept customer deposits. They provide loans to other

[8]A natural extension of the model is to combine agents in order to create bank holding company representations with realistic balance sheet structures that operate a combination of the commercial bank and investment bank business model. Each entity of such a hybrid agent would act according to its own business model, while profits, losses and information would be shared with all other entities organized under the same holding company. Regulation would apply to the balance sheet at the holding company level instead of at the entity level.

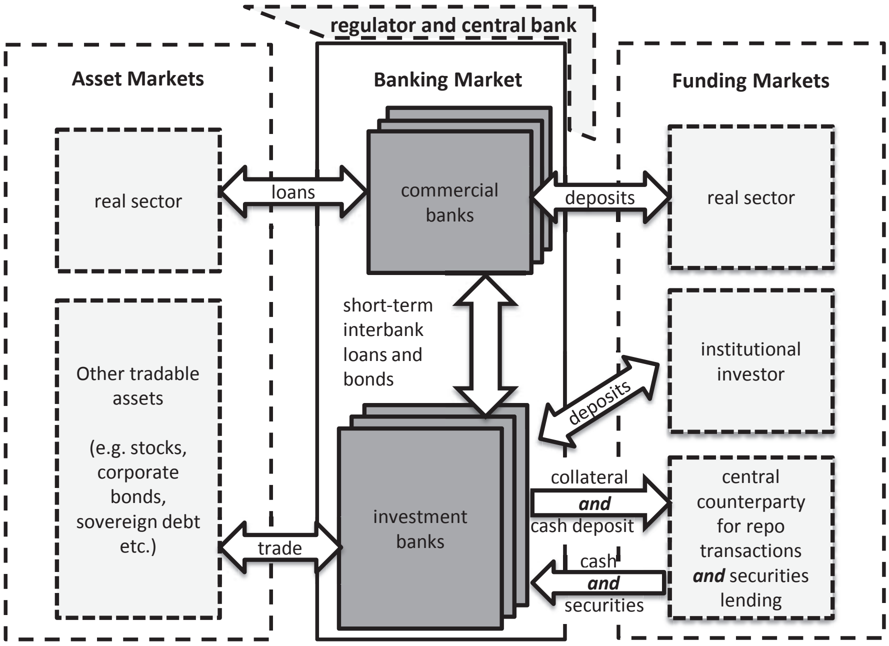


Figure 1: Model setup.

banks and specialize in trading on secondary asset markets. For funding they rely on equity, unsecured (and therefore volatile) deposits from institutional investors, short sales and repurchase agreements (repos), in which cash is provided in exchange for collateral by a central counterparty. The investment bank agents in our model do not directly lend to each other. Nevertheless, they are indirectly highly interconnected through overlapping portfolios. Given the features of their business model, investment bank agents may also represent financial institutions that are often classified as shadow banks (e.g. hedge funds, money market funds, structured investment vehicles etc.).

3.1 Commercial Banks

Commercial banks in the model issue loans to the real sector, which they fund via equity and different types of debt. Thereby, they profit from the spread between interest rates on loans and refinancing costs. The balance sheet of commercial bank agent $c \in \{1, 2, ..., n^C\}$ at the end of period t has the following structure

Assets	Liabilities	
Loans, $L_{c,t}$	Deposits, $D_{c,t}$	
Cash, $C_{c,t}$	Short-term Debt, $I_{c,t}$	$W_{c,t}$
	Long-term Debt, $B_{c,t}$	
	Equity, $E_{c,t}$	

with $W_{c,t}$ denoting wholesale debt.

It is important to consider the timing of the commercial bank model in order to derive the laws of motion for the balance sheet variables. Each period starts with the registration of defaults on loans and changes to deposits. To simplify matters, we assume occur overnight, i.e. between the end of period $t-1$ and the beginning of period t. Agents furthermore pay and receive interest payments as well as principal payments at the beginning of a period. The first decision entails the appropriation of profits that have accrued overnight. They can either be retained or paid out as dividends to shareholders. Decisions on the issuance of new loans and desired cash holdings follow. Ultimately, agents raise funds on wholesale debt markets in order to equilibrate the asset and liability side of their balance sheets.

The profit of commercial bank agent c that needs to be allocated in period t can be computed from the difference between interest payments on loans and the last period's funding costs:

$$\Pi_{c,t} = L_{c,t-1} r^L_{c,t} - (D_{c,t-1} + W_{c,t-1}) r^O_{c,t-1}, \tag{3.1}$$

with $r^O_{c,t} = (D_{c,t} r^D_{c,t} + B_{c,t} \bar{r}^B_{c,t} + I_{c,t} \bar{r}^I_{c,t})/(D_{c,t} + W_{c,t})$ being a composite measure of funding costs comprising three components: the interest rate paid for deposits $r^D_{c,t}$, the average interest rate paid for long term wholesale debt $\bar{r}^B_{c,t}$ and the average interest rate paid for short term debt $\bar{r}^I_{c,t}$, which besides interbank interest rates includes borrowing from the marginal lending facility of the central bank. The effective return on loans $r^L_{c,t}$ is a function of the charged interest rate $\tilde{r}^L_{c,t-1}$ and the stochastic process $\rho^L_{c,t}$ of loan default rates:

$$r^L_{c,t} = (1 - \rho^L_{c,t}) \tilde{r}^L_{c,t-1} - \rho^L_{c,t} \tag{3.2}$$

Both $\tilde{r}^L_{c,t}$ and $\rho^L_{c,t}$ are exogenous to our model.

The decision on the appropriation of profit follows a simple rule: We assume that commercial bank agents have an exogenously defined equity target E^*_c and that they cannot raise new equity from outside sources. As a consequence, any profits $\Pi_{c,t}$ that would lead to equity above the target are paid out as dividends $Div_{c,t}$:[9]

$$\begin{aligned} E_{c,t} &= \min\{E^*_c, E_{c,t-1} + \Pi_{c,t}\}, \\ Div_{c,t} &= \max\{0, E_{c,t-1} + \Pi_{c,t} - E_{c,t}\} \end{aligned} \tag{3.3}$$

The difference between profit and dividends is defined as the agent's retained earnings, i.e. $\Delta\Pi_{c,t} = \Pi_{c,t} - Div_{c,t}$. According to the timing of the model explained above, retained earnings, changes to deposits, incoming and outgoing debt payments change the cash holdings before any asset side decisions are considered by commercial bank agents. Taking this into account, we define an intermediate measure for cash holdings:

$$C'_{c,t} = C_{c,t-1} + \Delta\Pi_{c,t} + \Delta D_{c,t} + (1 - \rho^L_{c,t})(1 - m_L) L_{c,t-1} - (1 - m_B) B_{c,t-1} - I_{c,t-1}, \tag{3.4}$$

with $\Delta D_{c,t}$ being a random change in customer deposits, $(1 - m_L)$ being the constant repayment rate for performing loans and $(1 - m_B)$ being the constant repayment rate for long term wholesale debt.[10] The parameters $\{m_L, m_B\} \in [0, 1]$ can be interpreted as a maturity parameter for loans and long term debt respectively, with $\{m_L, m_B\} = 0$ meaning that debtors need to repay all credit every period, while $\{m_L, m_B\} = 1$ implies that the principal of loans is never to be paid

[9]Our heuristic for the payout management of bank agents is supported by empirical evidence. Adrian et al. (2015) e.g. find that financial institutions adjust payouts in order to achieve a desired path for equity. Furthermore, in times of financial stress, which is at the focus of the analysis conducted with the model, it can be rather difficult or undesirable to raise equity capital.

[10]In reality, loans to the real sector and long term wholesale debt typically do not exhibit constant repayment rates. However, when interpreting $L_{c,t}$ and $B_{c,t}$ as portfolios of loans and debt contracts, the modeling choice becomes more realistic.

back. Taking into account that $(1-m_L)m_L^{t-1}$ yields the fraction of the loan portfolio that matures at date t, we can compute the average maturity of the loan portfolio as follows:

$$\int_1^\infty (1-m_L)m_L^{t-1}t\delta t = \frac{(m_L-1)(\log(m_L)-1)}{\log(m_L)^2}. \tag{3.5}$$

Due to regulatory or preference related reasons, it may be in the interest of commercial bank agents to hold a certain amount of cash (liquid assets) at the end of period t. The law of motion for cash holdings therefore encompasses the choice variable $\Delta C_{c,t}$:

$$C_{c,t} = C'_{c,t} + \Delta C_{c,t} \tag{3.6}$$

Likewise, the law of motion for loans to the real sector on a commercial bank agent's balance sheet contains a choice on how many new loans should be issued:

$$L_{c,t} = m_L L_{c,t-1}(1-\rho^L_{c,t}) + \Delta L_{c,t} \tag{3.7}$$

While loans and cash positions depend on decisions $\Delta L_{c,t}$ and $\Delta C_{c,t}$, equity follows a fixed rule and deposits fluctuate randomly around their initial value $D_{c,0}$ (i.e. $D_{c,t} = D_{c,0} + \Delta D_{c,t}$), the change to the volume of wholesale debt $\Delta W_{c,t}$ is derived from the balance sheet identity:

$$W_{c,t} = W_{c,t-1} + \Delta W_{c,t} = L_{c,t} + C_{c,t} - E_{c,t} - D_{c,t} \tag{3.8}$$

Therefore, the funding decision a commercial bank agent faces concerns the composition of wholesale debt rather than its volume. Specifically, each agent will have to choose the fraction $a_{c,t}$ of wholesale debt to be borrowed in long term debt, i.e.

$$B_{c,t} = a_{c,t}W_{c,t}. \tag{3.9}$$

The amount of short term funding is simply the difference between wholesale debt and long term debt, i.e. $I_{c,t} = W_{c,t} - B_{c,t}$. Both short term and long term debt will be raised from investment banks (agents $i \in \{1, 2, ..., n^I\}$), or - in the case of a shortage of credit supply - from the central bank's (agent CB's) marginal lending facility. For the sake of simplicity, we assume that funds from the central bank as well as short term interbank debt need to be rolled over each period. We can therefore define the total volume of short term debt as the sum of interbank short term debt and funds from the central bank: $I_{c,t} = I_{c,CB,t} + \sum_{i=1}^{n^I} I_{c,i,t}$.

3.1.1 Asset Side Management

In managing the asset side of their balance sheet, commercial bank agents follow a simple behavioral rule: expand the balance sheet if risk management does not object and the expected return on loans $\mathrm{E}_{c,t}[r^L]$ exceeds the expected marginal wholesale refinancing costs $\mathrm{E}_{c,t}[r^{\Delta W}]$. $\mathrm{E}_{c,t}[\cdot]$ denotes the expectations operator, with the subindices defining by whom and in what period expectations are formed. Taking into account the two conditions of the asset side management heuristic, the targeted volume of newly issued loans $\Delta L^*_{c,t}$ amounts to

$$\Delta L^*_{c,t} = \begin{cases} \Delta L^{risk}_{c,t} & \text{if} \quad \mathrm{E}_{c,t}[r^L] \geq \mathrm{E}_{c,t}[r^{\Delta W}] \\ \max(0, E_{c,t} + D_{c,t} - m_L L_{c,t-1}(1-\rho^L_{c,t})) & \text{if} \quad \mathrm{E}_{c,t}[r^L] < \mathrm{E}_{c,t}[r^{\Delta W}]. \end{cases} \tag{3.10}$$

Equation (3.10) introduces commercial bank agents' risk management, with $\Delta L^{risk}_{c,t}$ specifying an upper limit for newly issued loans for which risk seems acceptable. The amount of loans on the balance sheet will be bound by available equity and deposits if the cost of wholesale funding exceeds returns on loans. Thereby we assume that the cost for deposits are always lower than the return on loans, i.e. $r^D < r^L$.

Commercial bank agents employ a value at risk approach to specify the risk-management-bound $\Delta L_{c,t}^{risk}$ for new loans. The bound seeks to ensure that the bank's equity is sufficient to absorb losses from defaults on loans and variations in refinancing costs. Technically, risk management allows the issuance of new loans only until the bank's total value at risk is equal to its equity, i.e. $\Delta L_{c,t}^{risk} = VaR_t^{-1}(-E_{c,t})$. A bank's value at risk is computed as the sum of the values at risk with confidence level x^L from outstanding loans VaR_{out}, prospective loans VaR_{prosp} and wholesale refinancing costs VaR_{ref}:[11]

$$VaR_t(\Delta L) = VaR_t^{out} + VaR_t^{prosp}(\Delta L) + VaR_t^{ref}(\Delta L, \Delta C) \tag{3.11}$$

with

$$VaR_t^{out} = \mathop{\mathrm{E}}_{c,t}\left[F_{out}^{-1}(x^L)\right] m_L L_{t-1}(1-\rho_{c,t}), \tag{3.12}$$

$$VaR_t^{prosp}(\Delta L) = \mathop{\mathrm{E}}_{c,t}\left[F_{prosp}^{-1}(x^L)\right] \Delta L \quad \text{and} \tag{3.13}$$

$$VaR_t^{ref}(\Delta L, \Delta C) = \mathop{\mathrm{E}}_{c,t}\left[F_{ref}^{-1}(x^L)\right] W_{c,t}(\Delta L, \Delta C). \tag{3.14}$$

F_{out}^{-1}, F_{prosp}^{-1} and F_{ref}^{-1} are the inverse cumulative distribution functions, or quantile functions, of losses from outstanding loans, losses from prospective loans and refinancing costs, respectively. The expected values of the respective quantile functions at point x^L are computed through Monte Carlo simulations described in Section 3.1.4. Although such simulations increase computation time, they allow us to include realistic assumptions about the stochastic process of loan defaults into the risk management of agents. An analytical derivation of the loss distribution under realistic assumptions is often not possible.

In the context of our model, there is no intrinsic reason for commercial bank agents to hold cash at the end of a period. In order to minimize their need for external funding, agents will target a change in cash holdings equal in magnitude but opposite in sign to the current (intermediate) cash holdings, i.e. $\Delta C_{c,t}^* = -C'_{c,t}$. Nevertheless, there are two conditions under which cash holdings at the end of a period will be positive instead of zero. First, regulatory requirements such as a liquidity coverage ratio may impose a bound on cash holdings in dependence on the loan portfolio and other balance sheet variables, i.e. $\Delta C_{c,t}^{reg}(\cdot) = C_{c,t}^{reg}(\cdot) - C'_{c,t}$. Second, a commercial bank agent may want to deleverage faster than the maturity structure of its debt permits. In this case, proceeds from loans will be held in cash until they can be used to pay back long term debt. Since the maturity of long term debt imposes a lower limit to the wholesale debt volume (i.e. $W_{c,t-1} + \Delta W_{c,t} \geq m_B B_{c,t-1}$), the deleveraging bound for changes in cash amounts to $\Delta C_{c,t}^{del}(L) = m_B B_{c,t-1} + D_{c,t} + E_{c,t} - C'_{c,t} - L$. The constraints on cash are thus defined by two lower bounds: $\Delta C_{c,t} \geq \{\Delta C_{c,t}^{reg}, \Delta C_{c,t}^{del}\}$. Unless the target $\Delta C_{c,t}^*$ is smaller than one of these bounds, $\Delta C_{c,t} = \Delta C_{c,t}^*$. Otherwise, $\Delta C_{c,t}$ is set to the value of the violated bound. Lending is constrained from above by a precautionary limit and by regulation: $\Delta L_{c,t} \leq \{\Delta L_{c,t}^{prec}, \Delta L_{c,t}^{reg}\}$. The precautionary limit $\Delta L_{c,t}^{prec} := (1 - m_L)L_{c,t-1} + E_{c,t}$ inhibits an excessively rapid build up of the loan portfolio. In general, if $L_{c,t}^{reg}(\cdot)$ defines the maximum volume of loans permitted by the regulator given the state of relevant balance sheet variables, the regulatory upper bound amounts to $\Delta L_{c,t}^{reg}(\cdot) = L_{c,t}^{reg}(\cdot) - m_L L_{c,t-1}(1 - \rho_{c,t})$.[12]

[11] For the sake of simplicity, we assume perfect positive correlation between the individual risks in Eq.(3.11) by modeling the total value of risk as the unweighted sum of individual value of risks. VaR is therefore the upper bound for the true value of risk.

[12] A regulatory upper bound on the issuance of new loans to the real sector may seem arbitrary. However, since

3.1.2 Liability Side Management

The total volume of wholesale debt to be raised by a commercial bank agent is determined by the asset side management and the law of motion specified in Eq. (3.8). The task of an agent's liability side management therefore boils down to the decision on what proportion $a_{c,t} \in [0,1]$ of its wholesale debt volume $W_{c,t}$ should be borrowed in long term debt $B_{c,t} = a_{c,t}W_{c,t}$. Both, the interest rate for new short term debt $r^I_{c,t}$ and new long term debt $r^B_{c,t}$ are endogenous to our model. They are set each period by individual investment bank agents and a market maker, respectively. The structural difference between the two forms of wholesale debt is defined by the exogenously set maturity parameter m_B. While short term debt is raised at the end of period $t-1$ from individual investment bank agents and needs to be fully repaid at the beginning of period t, long term debt is issued (like a bond) at the end of period $t-1$ and only a fraction $(1-m_B)$ of it matures at the beginning of period t. Hence, for commercial bank agent c the cost for short term debt in period t is a weighted sum of interest rates determined in period t:

$$r^I_{c,t} = \frac{\sum_{i=1}^{n^I} I_{c,i,t} r^I_{i,c,t}}{\sum_{i=1}^{n^I} I_{c,i,t}}, \tag{3.15}$$

with $I_{c,i,t}$ denoting the volume of short term debt borrowed from investment bank agent i and $r^I_{i,c,t}$ being the interest rate charged by that investment bank agent. In the case of insufficient supply of short term interbank debt, commercial bank agents will have to resort to borrowing from the central bank, which offers limitless credit from its marginal lending facility at a relatively expensive interest rate r^{CB}_t. With $I_{c,CB,t} = I_{c,t} - \sum_{i=1}^{n^I} I_{c,i,t}$ being the demand for central bank credit and $a^{CB}_{c,t} = I_{c,CB,t}/I_{c,t}$ being the fraction of short term debt borrowed from the central bank, the average interest rate for short term debt amounts to

$$\bar{r}^I_{c,t} = (1 - a^{CB}_{c,t})r^I_{c,t} + a^{CB}_{c,t} r^{CB}_t. \tag{3.16}$$

The average short term interest rate is potentially very volatile, since both the fraction of short term debt borrowed from the central bank as well as the interest rate charged by investment bank agents may change every period. The cost for long term debt at time t is typically less volatile, since it is calculated as a weighted sum of past and current interest rates:

$$\bar{r}^B_{c,t} = \frac{m_B B_{c,t-1}\bar{r}^B_{c,t-1} + \Delta B_{c,t} r^B_{c,t}}{B_{c,t}}, \tag{3.17}$$

with $\bar{r}^B_{c,t}$ denoting the current average interest rate for long term debt and $\Delta B_{c,t} = B_{c,t} - m_B B_{c,t-1}$ being the newly borrowed long term funds. The maturity parameter m_B adds persistence to long term funding costs. We infer that if commercial bank agents are risk averse, and we assume they are, there exists a trade-off between funding costs and funding stability from which an optimal share $a^*_{c,t}$ of long term debt can be derived. We model this trade-off through a mean-variance optimization of the expected interest rate surplus in the next period $S_{c,t+1}$ per unit of outstanding loans this period:

$$a^*_{c,t} = \arg\max_a \left(\mathrm{E}_{c,t}[S_{t+1}] - 0.5\lambda_{c,t}\mathrm{Var}_{c,t}(S_{t+1})\right), \tag{3.18}$$

agents lack the ability to raise equity quickly, any capital requirements regulation can be translated into a limit on the volume of assets on the balance sheet.

with $\lambda_{c,t}$ being a time-variant scaling factor[13] and

$$S_{c,t+1} = r^L_{c,t+1} - \frac{W_{c,t}}{L_{c,t}} \left(a_{c,t}\bar{r}^B_{c,t+1} + (1 - a_{c,t})\bar{r}^I_{c,t+1} \right). \tag{3.19}$$

Since $a^*_{c,t}$ is a target ratio, we assume that it is calculated under the hypothetical premise that the composition of wholesale debt can be freely chosen in period t and that all maturing debt needs to refinanced in period $t+1$. In this case the average long term interest rate in period $t+1$ adds up to $\bar{r}^B_{c,t+1} = m_B r^B_{c,t} + (1 - m_B) r^B_{c,t+1}$. When covariance terms are neglected, the solution to the maximization problem in Eq. (3.18) can be approximated with the following equation (see Appendix B):

$$a^*_{c,t} \approx \frac{\hat{\mathrm{Var}}_{c,t}(\bar{r}^I, \psi^I) - \frac{\mathrm{E}_{c,t}[r^B] - \mathrm{E}_{c,t}[\bar{r}^I]}{\lambda_{c,t}} \frac{L_{c,t}}{W_{c,t}}}{\hat{\mathrm{Var}}_{c,t}(\bar{r}^I, \psi^I) + (1 - m_B)^2 \hat{\mathrm{Var}}_{c,t}(r^B, \psi^B)} \tag{3.20}$$

The operator $\hat{\mathrm{Var}}(x, \psi)$ denotes an exponentially weighted moving variance with memory parameter ψ. A detailed derivation of expected values and variances is given in Section 3.1.4. Three factors may interfere with a commercial bank agent's ability to reach its desired composition of wholesale debt. First and foremost, the supply of long term debt poses an upper bound to the desired composition, i.e. $a^{\sup}_{c,t} = \sum_{i=1}^{n^I} (m_B B_{c,t-1} + \Delta B_{i,c,t} - B^{MM}_{c,t-1})/W_{c,t}$, with $\Delta B_{i,c,t}$ being the excess supply of long term funding from investment bank agent i to commercial bank agent c and $B^{MM}_{c,t-1}$ being the value of the market makers inventory of loans to that commercial bank. Second, because only the maturing long term debt $(1 - m_B)B_{c,t-1}$ can be repaid or replaced by other forms of debt, there will be a maturity induced lower bound $a^{mat}_{c,t} = m_B B_{c,t-1}/W_{c,t}$ to the proportion of wholesale debt borrowed in long term debt. Third, regulatory requirements may impose a limit $a^{reg}_{c,t}$ to the amount of short term debt on a bank's balance sheet. Therefore $a_{c,t}$ has an upper and two lower bounds, i.e. $a^{\sup}_{c,t} \geq a_{c,t} \geq \{a^{reg}_{c,t}, a^{mat}_{c,t}\}$. If no bound is violated the share of long term debt is set to the target rate $a^*_{c,t}$. With the ratio of long term to short term wholesale debt, the average $r^W_{c,t}$ and marginal $r^{\Delta W}_{c,t}$ wholesale refinancing costs can be computed:

$$r^W_{c,t} = a_{c,t}\bar{r}^B_{c,t} + (1 - a_{c,t})\bar{r}^I_{c,t} \tag{3.21}$$

$$r^{\Delta W}_{c,t} = a_{c,t} r^B_{c,t} + (1 - a_{c,t})\bar{r}^I_{c,t} \tag{3.22}$$

Note that the marginal wholesale refinancing costs consider the current interest rate for long term debt instead of the average interest rates currently paid for long term loans.

3.1.3 Raising short term interbank debt

Once the demand for short term interbank loans $I_{c,t}$ is known to the commercial bank agent, it needs to raise these funds on the interbank market. This involves two steps: First, offers from the interbank market are collected and evaluated. Loan offers will be provided by all n^I investment bank agents and a central bank. The central bank acts as a lender of last resort (i.e. $I_{CB,c,t} = \infty$) and ensures that sufficient funding is available to commercial bank agents. Second, starting with the offer with the best valuation, agents engage in bilateral transactions until their

[13] In a mean-variance optimization setup, λ would typically be a risk aversion parameter. However, in our context it makes sense to define λ as a variable scaling factor. This will allow us to calibrate the trade-off between funding costs and funding stability along the lines of the following statement: if the probability that short term funding costs exceed long term funding costs is smaller than x percent, then wholesale debt should be exclusively short term.

demand for short term interbank funding is completely satisfied. According to this procedure, the loan received from investment bank i amounts to

$$I_{c,i,t} = \begin{cases} I_{i,c,t} & \text{if } I_{c,t} - I_{i,c,t} - \sum_{\iota \in \mathcal{U}} I_{\iota,c,t} \geq 0 \\ I_{c,t} - \sum_{\iota \in \mathcal{U}} I_{\iota,c,t} & \text{if } I_{i,c,t} > I_{c,t} - \sum_{\iota \in \mathcal{U}} I_{\iota,c,t} \\ 0 & \text{else} \end{cases} \tag{3.23}$$

with $I_{i,c,t}$ (note the switched order of subindices) denoting the loan volume offered by investment bank agent i and $\mathcal{U} := \{\iota | U^C_{\iota,c,t} > U^C_{i,c,t}\}$ being the set of offers with a higher valuation U^C than the offer from investment bank agent i. We assume that an offer is evaluated according to two factors: the trust $v_{c,i,t}$ between commercial bank agent c and investment bank agent i and the relative attractiveness $u_{c,i,t}$ of the interest rate. The two factors are evaluated jointly via a Cobb-Douglas function

$$U^C_{c,i,t} = v^{\gamma_v}_{c,i,t} u^{\gamma_u}_{c,i,t}, \tag{3.24}$$

with the exponents γ_v and γ_u being the valuation elasticities of the two factors. The trust component is motivated by relatively recent empirical evidence showing that the frequency of past transactions between two parties is a good indicator for current links in the interbank market (see e.g. Cocco et al., 2009; Finger and Lux, 2014; Craig et al., 2015). The variable

$$\xi_{i,c,t} = \begin{cases} 1 & \text{if a transaction takes place} \\ -1 & \text{if no transaction takes place} \end{cases}$$

indicates whether a transaction between commercial bank agent c and investment bank agent i has taken place in period t or not, while the variable

$$\Xi_{c,i,t} = \begin{cases} \Xi^{max} & \text{if } \Xi_{c,i,t-1} + \frac{\xi_{i,c,t-1}}{\Xi^{max}} \geq \Xi^{max} \\ \Xi^{min} & \text{if } \Xi_{c,i,t-1} + \frac{\xi_{i,c,t-1}}{\Xi^{max}} \leq \Xi^{min} \\ \Xi_{c,i,t-1} + \frac{\xi_{i,c,t-1}}{\Xi^{max}} & \text{else} \end{cases}$$

defines the aggregation mechanism for the transaction indicator. Within the range of permissible values, trust $v_{c,i,t} \in [\Xi^{min}/\Xi^{max}, 1]$ increases when agents engage in a transaction and decreases otherwise:

$$v_{c,i,t} = \frac{\Xi_{c,i,t}}{\Xi^{max}} \tag{3.25}$$

The parameter $\Xi^{max} > 1$ defines the stickiness with which trust increases or decreases. The larger Ξ^{max}, the more transactions are necessary before two agents completely trust each other. The relative attractiveness of the interbank interest rate $r^I_{i,c,t}$ demanded by investment bank agent i for a loan to commercial bank agent c is straightforward. The closer the interest rate is to the currently lowest demanded rate $r^{low}_{c,t}$, the higher its attractiveness:

$$u_{i,c,t} = \frac{r^{low}_{c,t}}{r^I_{i,c,t}} \tag{3.26}$$

3.1.4 Expectation Formation

Commercial bank agents need to form expectations about all stochastic variables and processes. This includes the effective return on loans, which with Equation (3.2) amounts to $\mathrm{E}_{c,t}[r^L] = (1 - \mathrm{E}_{c,t}[\rho^L])\tilde{r}^L_{c,t-1} - \mathrm{E}_{c,t}[\rho^L]$. For the sake of simplicity, we assume that commercial bank agents know the exogenous stochastic process of the loan default rate $\rho^L_{c,t}$ and can compute its moments, i.e. $\mathrm{E}_{c,t}[\rho^L] = \mu^{\rho^L}_{c,t}$ and $\sqrt{\mathrm{Var}_{c,t}(\rho^L)} = \sigma^{\rho^L}_{c,t}$. Under this assumption, we can derive estimates of the value at risk contributions (see Eq. (3.12) and (3.13)) of outstanding and prospective portfolio losses. Specifically, agents obtain empirical loss-distributions by simulating n^{risk} evolutions of their loan portfolio. In the l-th simulation by commercial bank agent c in period t, the relative volume of outstanding loans $\check{L}^{out}$ evolves as follows:

$$\check{L}^{out}_{c,l,t+\tau} = \prod_{x=1}^{\tau} (1 - \check{\rho}^{L_{out}}_{c,l,t+x}) m^x_L, \tag{3.27}$$

with $\check{\rho}^{L_{out}}$ being random values generated with the known stochastic process of the default rate of outstanding loans. The corresponding loss rate $\Lambda^{out}_{c,l,t+T^{risk}}$ in period T^{risk} amounts to

$$\Lambda^{out}_{c,l,t+T^{risk}} = \sum_{\tau=1}^{T^{risk}} (\check{\rho}^{L_{out}}_{c,l,t+\tau} - (1 - \check{\rho}^{L_{out}}_{c,l,t+\tau})\tilde{r}^{L_{out}}_{c,l,t})\check{L}^{out}_{c,l,t+\tau-1}. \tag{3.28}$$

Because loans in our model are never fully paid back we arbitrarily choose $T^{risk} = \log(0.01)/\log(m_L)$ (the period in which 99% of loans have been repaid) as the simulation length. We furthermore define $\mathcal{L}^{out}_{c,t}(l)$ as the l-th element of the ordered set of losses from the portfolio of outstanding loans after T^{risk} periods:

$$\mathcal{L}^{out}_{c,t} := \{\Lambda^{out}_{c,1,t+T^{risk}}, \Lambda^{out}_{c,2,t+T^{risk}}, ..., \Lambda^{out}_{c,n^{risk},t+T^{risk}}\}, \tag{3.29}$$

with the elements of the set ordered ascendingly, i.e. $\mathcal{L}^{out}_{c,t}(l) \leq \mathcal{L}^{out}_{c,t}(l+1)$. The estimates of the quantile functions of losses from outstanding loans (losses from prospective loans are derived analogously) at point x^L can be expressed as

$$\mathop{\mathrm{E}}_{c,t}[F^{-1}_{out}(x^L)] = \mathcal{L}^{out}_{c,t}(x^L n^{risk}) \quad \text{and} \quad \mathop{\mathrm{E}}_{c,t}[F^{-1}_{prosp}(x^L)] = \mathcal{L}^{prosp}_{c,t}(x^L n^{risk}). \tag{3.30}$$

Unlike the stochastic process generating loan defaults, wholesale refinancing costs are endogenous to our model. For the sake of simplicity, the risk management of commercial bank agents model wholesale refinancing costs in consecutive periods as i.i.d. normally distributed random variables. This allows us to derive the expected value $\mathrm{E}_{c,t}[r^W_{total}]$ and variance $\mathrm{Var}_{c,t}(r^W_{total})$ of the total wholesale refinancing costs of a loan portfolio analytically:

$$\mathop{\mathrm{E}}_{c,t}[r^W_{total}] = \sum_{\tau=0}^{\infty} m^\tau_L \hat{\mathrm{E}}_{c,t}[r^W, \psi^W] = \frac{\hat{\mathrm{E}}_{c,t}[r^W, \psi^W]}{1 - m_L} \tag{3.31}$$

$$\mathop{\mathrm{Var}}_{c,t}(r^W_{total}) = \sum_{\tau=0}^{\infty} m^{2\tau}_L \hat{\mathrm{Var}}_{c,t}(r^W, \psi^W) = \frac{\hat{\mathrm{Var}}_{c,t}(r^W, \psi^W)}{1 - m^2_L}. \tag{3.32}$$

Note that by discounting wholesale cost with the maturity parameter of loans m_L, we assume that the need for wholesale funding and the volume of outstanding loans to the real sector decrease at the same speed. This is a cautious assumption since loans are funded partly by customer deposits and equity. Given the quantile function of a normally distributed random variable, we compute

$$\underset{c,t}{\mathrm{E}}[F_{ref}^{-1}(x^L)] = \underset{c,t}{\mathrm{E}}[r_{total}^W] + \sqrt{2\underset{c,t}{\mathrm{Var}}(r_{total}^W)}\,\mathrm{inverf}(2x^L - 1), \tag{3.33}$$

with inverf$(\cdot)$ being the inverse of the error function. The operators $\hat{\mathrm{E}}_t[x,\psi] := (1-\psi)\hat{\mathrm{E}}_{t-1}[x] + \psi x_t$ in Eq. (3.31) and $\hat{\mathrm{Var}}_t(x,\psi) := \hat{\mathrm{E}}_t\left[(x - \hat{\mathrm{E}}_t[x,\psi])^2, \psi\right]$ in Eq. (3.32) compute the exponentially weighted moving average and variance of variable x respectively, with ψ being a memory parameter determining the weight of the latest observation of x. By using exponentially weighted moving averages and variances, we can control how quickly past events become irrelevant to present decision making.

In the context of our model, it seems reasonable to assume that the current short term and long term interest rates, which are endogenously determined, reflect all available information. Therefore, commercial bank agents expect the future rate to be the current rate, i.e. $\mathrm{E}_{c,t}[r^B] = r^B_{c,t}$ and $\mathrm{E}_{c,t}[r^I] = r^I_{c,t}$. The expected value of the average short term interest paid $\bar{r}^I_{i,t}$ takes into account the possibility that a commercial bank agent's demand for short term debt is not completely met by investment bank agents. Since excess demand for short term debt will be covered by the lender of last resort, the expected value of the average short term interest rate amounts to:

$$\underset{c,t}{\mathrm{E}}[\bar{r}^I] = (1 - \mathrm{E}_{c,t}[a^{CB}])\underset{c,t}{\mathrm{E}}[r^I] + \mathrm{E}_{c,t}[a^{CB}]\underset{c,t}{\mathrm{E}}[r^{CB}], \tag{3.34}$$

with $a^{CB}_{c,t} = I_{c,CB,t}/I_{c,t}$ being the fraction of short term debt borrowed from the central bank at time t. Taking into account that the necessity for central bank funding may be erratic rather than smooth, the expected value of a^{CB} is modeled as an exponentially weighted moving average, i.e. $\mathrm{E}_{c,t}[a^{CB}] := \hat{\mathrm{E}}_{c,t}[a^{CB}, \psi^I]$. The expectation for the marginal wholesale refinancing cost are formed as follows:

$$\underset{c,t}{\mathrm{E}}[r^{\Delta W}] = \underset{c,t}{\mathrm{E}}[a]\underset{c,t}{\mathrm{E}}[r^B] + (1 - \underset{c,t}{\mathrm{E}}[a])\underset{c,t}{\mathrm{E}}[\bar{r}^I] \tag{3.35}$$

Note that commercial bank agents need to form expectations about the ratio $a_{c,t}$ of wholesale funding received in long term debt. This is due to the circular dependency between expected marginal wholesale refinancing costs and $a_{c,t}$.[14] In order to resolve this circular dependency, the expected ratio of wholesale funding received in the form of long term debt is defined as the ratio computed under the assumption that agents ignore the refinancing constraint in Eq. (3.10), i.e. $\mathrm{E}_{c,t}[a] := a_{c,t}|_{\Delta L_{c,t} = \min\{\Delta L^{risk}_{c,t}, \Delta L^{prec}_{c,t}, \Delta L^{reg}_{c,t}\}}$. The assumption implies a temporal order of events within the decision making process of a bank: First, a loan officer of the bank communicates the volume of loans she wants to issue in a given period. Second, the risk management department checks whether the potential investment is acceptable in terms of risk. Third, the funding department computes the associated funding costs. Fourth, the loan officer collects the information from the respective departments, compares funding costs with the expected return on the investment and decides whether the loan will be granted or not.

Each period maturing long term debt needs to be refinanced. Commercial bank agents therefore worry about the volatility of long term interest rates, which is computed as the exponentially weighted moving variance:

$$\hat{\mathrm{Var}}_{c,t}(r^B) = \hat{\mathrm{E}}_{c,t}\left[(r^B - \hat{\mathrm{E}}_{c,t}[r^B, \psi^B])^2, \psi^B\right], \tag{3.36}$$

[14] Via Eqs. (3.18), (3.10) and (3.35) it becomes clear that $\mathrm{E}_{c,t}[r^{\Delta W}]$ is needed to obtain $\Delta L_{c,t}$, which is needed to compute $a_{c,t}$, which in turn is an input to $\mathrm{E}_{c,t}[r^{\Delta W}]$.

Assuming, for the sake of simplicity, a constant interest rate r^{CB} of the central bank's marginal lending facility and that $a^{CB}_{c,t}$ and $r^I_{c,t}$ are independent, the variance of the average short term interest rate amounts to

$$\begin{aligned}\hat{\mathrm{Var}}_{c,t}(\bar{r}^I, \psi^I) &= \hat{\mathrm{E}}_{c,t}\left[(\bar{r}^I - \hat{\mathrm{E}}_{c,t}[\bar{r}^I, \psi^I])^2, \psi^I\right] \\ &= \hat{\mathrm{E}}_{c,t}\left[(a^{CB} - \hat{\mathrm{E}}_{c,t}[a^{CB}, \psi^I])^2 (r^{CB} - r^I)^2, \psi^I\right]. \end{aligned} \tag{3.37}$$

3.2 Investment Banks

In the model, investment banks form expectations about the returns and risks of three different kinds of assets: interbank loans, bank bonds and non-bank debt securities (nb-securities). The maturity of nb-securities is specified by the parameter $m_S \in [0,1]$. While short term interbank loans have a maturity of one day, only a fraction $(1 - m_B)$ of bank bonds mature each period. In essence, bank bonds are tradable securitized long term loans to commercial bank agents. The investment bank agents finance their portfolio with equity, investor deposits, short term collateralized debt (repos) and by borrowing assets for the purpose of short selling. All nb-securities can be used as collateral in a repo transaction and borrowed for the purpose of short selling. The balance sheet of agent $i \in \{1, 2, ..., n^I\}$ at the end of period t has the following structure:

Assets		Liabilities	
Nb-securities,	$\sum_{s\in\mathcal{Q}^S_{\text{long}}} Q^S_{i,s,t} P^S_{s,t}$	Repos,	$\sum_{s=1}^{n^S} R_{i,s,t}$
Short-term Interbank Loans,	$\sum_{c=1}^{n^C} I_{c,i,t}$	Shorted Assets,	$\sum_{s\in\mathcal{Q}^S_{\text{short}}} \lvert Q^S_{i,s,t}\rvert P^S_{s,t}$
Bank bonds,	$\sum_{c=1}^{n^C} Q^B_{i,c,t} P^B_{c,t}$	Investor Deposits	$D_{i,t}$
Margin Account,	$\sum_{s=1}^{n^S} M_{i,s,t}$	Equity,	$E_{i,t}$
Cash,	$C_{i,t}$		

We define $Q^S_{i,s,t} \in \mathbb{R}$ as the quantity of nb-security s held by agent i at time t and $P^S_{s,t}$ as its current trading price. The quantity of an nb-security is negative when the asset is borrowed and sold short. Because a borrowed asset qualifies as debt, short and long positions of nb-securities need a different balance sheet treatment. Formally, $\mathcal{Q}^S_{\text{short}} := \{s \mid Q^S_{i,s,t} < 0\}$ defines the set of nb-securities that are shorted by agent i at time t and $\mathcal{Q}^S_{\text{long}} := \{s \mid Q^S_{i,s,t} \geq 0\}$ defines the set of nb-securities to which that agent has a positive exposure at time t. Note that in order to avoid that investment bank agents obtain unlimited funds through short sales, it is required that for each asset sold short agents deposit cash into a margin account on their own balance sheet. This is consistent with typical short selling regulation. Like nb-securities, bank bonds are described by a quantity $Q^B_{i,c,t} \geq 0$ and a price $P^B_{c,t}$. The investor deposits $D_{i,t}$, which can be used to fund investments in nb-securities, overnight interbank loans and bank bonds, are conceptually different from customer deposits in the commercial bank agent's balance sheet. Since investor deposits are not insured, increases and withdrawals are not random, but related to the profitability and risk of the corresponding investment bank agent.

The timing of the investment bank model is divided into two steps: First, principal and interest rate payments on interbank loans and securities are collected and the balance sheet is evaluated at market prices. Profits and losses are registered and dividends are paid out. Second, agents restructure their portfolio.

The balance sheet of investment bank agents evolves with the portfolio decisions described in the next section. Analogous to the modeling of commercial banks, investment bank agents have an equity target E_i^* and cannot raise new equity. Therefore,

$$E_{i,t} = \min\{E_i^*, E_{i,t-1} + \Pi_{i,t}\}. \tag{3.38}$$

Profits $\Pi_{i,t} = \Pi^I_{i,t} + \Pi^B_{i,t} + \Pi^S_{i,t} - Fund_{i,t}$ are generated through investments in short term interbank loans, bank bonds and nb-securities; they are reduced by the funding costs:

$$\Pi^I_{i,t} = \sum_{c\in\mathcal{S}} I_{c,i,t-1} r^I_{i,c,t-1} - \sum_{c\in\mathcal{S}^{-1}} I_{c,i,t-1} \tag{3.39}$$

$$\begin{aligned}\Pi^B_{i,t} =& \sum_{c\in\mathcal{S}} Q^B_{i,c,t-1}\left(\frac{B_{c,t-1}}{Q^B_{c,t-1}}\bar{r}^B_{c,t-1} + (1-m_B)(\frac{B_{c,t-1}}{Q^B_{c,t-1}} - P^B_{c,t-1}) + m_B(P^B_{c,t} - P^B_{c,t-1})\right)\\ &- \sum_{c\in\mathcal{S}^{-1}} Q^B_{i,c,t-1}P^B_{c,t-1}\end{aligned} \tag{3.40}$$

$$\Pi^S_{i,t} = \sum_{s=1}^{n^S} Q^S_{i,s,t-1}\left(\frac{V^S_{s,t}}{Q^S_s}\bar{r}^S_{s,t-1} + (1-m_S)(\frac{V^S_{s,t-1}}{Q^S_s} - P^S_{s,t-1}) + m_S(P^S_{s,t} - P^S_{s,t-1})\right) \tag{3.41}$$

$$Fund_{i,t} = r^D_{t-1}D_{i,t-1} + r^R_{t-1}R_{i,s,t-1} + r^M_{t-1}M_{i,s,t-1} \tag{3.42}$$

with $\mathcal{S} := \{c|E_{c,t} \geq 0\}$ and $\mathcal{S}^{-1} := \{c|E_{c,t} < 0\}$ defining the set of solvent and insolvent commercial bank agents, respectively. The value of outstanding long term debt ($B_{c,t}$) on the balance sheet of commercial bank agent c divided by the outstanding quantity of the corresponding bond ($Q^B_{c,t}$) is used to calculate the interest payments and loan repayments per bond. Note that while the total outstanding quantity of a bank bond Q^B is time dependent, the quantity Q^S is fixed. This means that all maturing shares of nb-securities have to be reissued each period. V^S defines the nominal value of outstanding nb-securities and $\bar{r}^S$ their nominal interest rate, both of which are exogenous. Funding costs depend on r^R_t, r^M_t and r^D_t, which denote the interest rate for repo transactions, short selling transactions and investor deposits, respectively.

3.2.1 Asset and Liability Management

Investment banks need to decide on the desired composition of their balance sheet each period. Their decision is formulated as a weight vector $\mathbf{a}_{i,t}$ which defines the desired value of the balance sheet items as a multiple of the value of current equity $E_{i,t}$. The column vector $\mathbf{a}_{i,t} = (\mathbf{a}^S_{i,t}, \mathbf{a}^B_{i,t}, \mathbf{a}^M_{i,t}, \mathbf{a}^R_{i,t}, a^D_{i,t}, a^I_{i,t}, a^C_{i,t})'$ contains four sub-vectors which comprise the individual weights of nb-securities $\mathbf{a}^S_{i,t} = (a^S_{i,1,t}, ..., a^S_{i,n^S,t})'$, bank bonds $\mathbf{a}^B_{i,t} = (a^B_{i,1,t}, ..., a^B_{i,n^C,t})'$, margin account deposits $\mathbf{a}^M_{i,t} = (a^M_{i,1,t}, ..., a^M_{i,n^S,t})'$, repo liabilities $\mathbf{a}^R_{i,t} = (a^R_{i,1,t}, ..., a^R_{i,n^S,t})'$ as well as weights for investor deposits $a^D_{i,t}$, a composite overnight interbank asset $a^I_{i,t}$ and cash $a^C_{i,t}$. The optimal weight for each balance sheet position is the solution to a mean-variance optimization problem (the solution algorithm is described in Appendix C):

$$\mathbf{a}^*_{i,t} = \arg\max_{\mathbf{a}} \mathbf{a}'\mathrm{E}_{i,t}[\mathbf{r}] - 0.5\lambda_i\mathbf{a}'\mathbf{\Sigma}_{i,t}\mathbf{a} \qquad \text{s.t.} \tag{3.43}$$

$$a^R_{i,s,t} = \begin{cases} -(1-h^R_{s,t})a^S_{i,s,t} & \text{if } a^S_{i,s,t} \geq 0 \text{ and } h^R_{s,t} \leq h^D_{i,t} \\ 0 & \text{else} \end{cases} \tag{3.44}$$

$$a^M_{i,s,t} = \begin{cases} -(1+k_{s,t})a^S_{i,s,t} & \text{if } a^S_{i,s,t} < 0 \\ 0 & \text{else} \end{cases} \tag{3.45}$$

$$a^D_{i,t} = -(1-h^D_{i,t})(a^I_{i,t} + \sum_{c=1}^{n^C} a^B_{i,c,t} + \sum_{s\in\mathcal{D}} a^S_{i,s,t}) \tag{3.46}$$

$$\{a^I_{i,t}, a^B_{i,c,t}, a^C_{i,t}\} \geq 0 \quad \text{and} \quad \mathbf{a}'\mathbf{1} = 1 \tag{3.47}$$

with $\mathbf{r} = (\mathbf{r}^S_{i,t}, \mathbf{r}^B_{i,t}, \mathbf{r}^M_{i,t}, \mathbf{r}^R_{i,t}, r^D_{i,t}, r^I_{i,t}, r^C_{i,t})'$ being a vector containing the returns of balance sheet positions, λ_i being the risk aversion parameter of investment bank i, $\mathbf{\Sigma_{i,t}}$ being agent i's estimate of the $N \times N$ variance-covariance matrix of asset returns ($N = 3n^S + n^C + 3$) and $\mathbf{1}$ denoting a $N \times 1$ vector of ones. While the expected returns of nb-securities, bonds and the composite interbank asset are defined in the next section, the expected returns of repos, margin accounts and investor deposits specify the respective transaction costs. The variance and covariance terms of the transaction costs are zero. The variables $h^R_{s,t}$ and $k_{s,t}$, which are derived in Section 3.3.2, are the haircut and margin requirement on repo and short-selling transactions, respectively. Section 3.3.3 explains how the funding provided by institutional investors in the form of deposits translates into the haircut $h^D_{i,t}$.[15] The constraints in Eqs. (3.44) and (3.45) introduce the interdependency of the nb-security weights and the corresponding repo, margin account and investor deposit weights. We assume that investment bank agents, which can choose to fund long positions either through repos or investor deposits, opt for the less restrictive debt form, i.e. the smaller haircut. The weight for investor deposits, defined in Eq. (3.46), thus needs to consider long positions in nb-securities for $s \in \mathcal{D}$ with $\mathcal{D} := \{s|(a^S_{i,s,t} \geq 0) \wedge (h^R_{s,t} > h^D_{i,t})\}$. Note that when investment bank agent i purchases an nb-security it automatically engages in a repo transaction or debt relationship with an investor where it receives $1 - h^R_{s,t}$ or $1 - h^D_{i,t}$ times the current value of asset s as a cash-loan from a central counterparty or institutional investors. These automatic debt relationship may cause the liabilities side of balance sheets to be larger than necessary. However, excess funding will be held in cash and will therefore not add any risk to agents' balance sheets. In a short selling transaction, the central counterparty will require the agent to deposit $1 + k_{s,t}$ times the current value of asset s in cash. This cash will be held in a margin account on the investment bank agent's own balance sheet. The constraints in Eq. (3.47) make sure that interbank assets, bonds and cash cannot be shorted and that the budget constraint is met.

From the vector of optimal weights $\mathbf{a}^*_{i,t}$ we can derive the balance sheet positions of nb-securities and their corresponding margin and repo accounts. The aspired quantity $Q^S_{i,s,t}$ for the asset s amounts to

$$Q^S_{i,s,t} = \frac{a^S_{i,s,t} E_{i,t}}{P^S_{s,t}}, \tag{3.48}$$

[15] In general, the haircut $h^R_{s,t}$ of a repo transaction is defined as the percentage difference between the value of one unit of collateral (i.e. the price of nb-security s) and the loan received in exchange for the collateral. This definition implies that the higher the haircut, the more equity capital is needed to finance the purchase of nb-security s. In effect, the haircut puts a limit to an agent's leverage: When the maximum volume $Q^{max}_s P_s$ of asset s is bought and repo-financed (i.e. $R^{max} = (1-h^R)Q^{max}_s P_s$), we can derive the maximum leverage possible with haircut h^R by considering the balance sheet identity $Q^{max}_s P_s = E + R^{max}_s$: $\text{lev}^{max} = R^{max}_s/E = 1/h^R - 1$. The concept of the haircut is therefore useful beyond the context of a repo transaction. It can be used to introduce capital requirements (both, in the form of risk weights and a leverage ratio) into the portfolio maximization problem of investment bank agents.

while demand for the asset is computed as follows: $\Delta Q^S_{i,s,t} = Q^S_{i,s,t} - m_S Q^S_{i,s,t-1}$, repos, margin account positions and investor deposits are given by $R_{i,s,t} = a^R_{i,s,t} E_{i,t}$, $M_{i,s,t} = a^M_{i,s,t} E_{i,t}$ and $D_{i,t} = a^D_{i,t} E_{i,t}$, respectively. Cash holdings $C_{i,t}$ are determined by two factors: They may result from the portfolio maximization problem and they may accumulate due to failed transactions in the interbank market and bond market. Therefore

$$C_{i,t} = a^C_{i,t} E_{i,t} + a^I_{i,t} E_{i,t} + \sum_{c=1}^{n^C} a^B_{i,c,t} E_{i,t} - \sum_{c=1}^{n^C} (I_{c,i,t} + Q^B_{i,c,t} P^B_{c,t}). \tag{3.49}$$

3.2.2 Interbank loans and bank bonds

From the weights vector $\mathbf{a}^B_{i,t}$ obtained by solving the portfolio optimization problem we compute the volume of funds investment bank agent i wants to invest in a bond issued by commercial bank agent c, i.e. $B_{i,c,t} = a^B_{i,c,t} E_{i,t}$. The market maker (see Section 3.3.5), which sets the price of bonds, will balance demand and supply when demand for outstanding bonds is insufficient. In case of excess demand (i.e. $\sum_{i=1}^{n^I} B_{i,c,t} > B_{c,t}$), the available bonds are allocated to investment bank agents proportional to their initial demand. The quantity of bonds on the balance sheet of agent i is thus defined as follows:

$$Q^B_{i,c,t} = \begin{cases} B_{i,c,t}/P^B_{c,t} & \text{if} \quad \sum_{i=1}^{n^I} B_{i,c,t} \leq B_{c,t} \\ \frac{B_{i,c,t}}{P^B_{c,t}} \frac{B_{c,t}}{\sum_{i=1}^{n^I} B_{i,c,t}} & \text{else} \end{cases} \tag{3.50}$$

The quantity of bonds investment bank agent i trades at time t is given by: $\Delta Q^B_{i,c,t} = Q^B_{i,c,t} - m_B Q^B_{i,c,t-1}$.

The allocation of short term interbank funds to individual commercial bank agents is derived by considering a combination of an evaluation function $U^I_{i,c,t}$ that discriminates between different commercial bank agents and a discrete choice model that transform evaluations into choices. First, the relevant factors are evaluated jointly with a Cobb-Douglas functions. These are the trust $v_{c,i,t}$ between the two agents as computed in Eq. (3.25), the expected returns $\mathrm{E}_{i,t}[r^I_c]$ and the expected standard deviations of returns $\sqrt{\mathrm{Var}_{i,t}(r^I_c)}$. Specifically,

$$U^I_{i,c,t} = \begin{cases} \left(\frac{v_{c,i,t}}{\max(v_{i,t})}\right)^{\gamma_{vi}} \left(\frac{\mathrm{E}_{i,t}[r^I_c]}{\max(\mathrm{E}_{i,t}[r^I_c])}\right)^{\gamma_r} \exp\left(\frac{-\sqrt{\mathrm{Var}_{i,t}(r^I_c)}}{\max(\sqrt{\mathrm{Var}_{i,t}(r^I_c)})}\right)^{\gamma_\sigma} & \text{if} \quad U^I_{i,c,t} > U^{\min} \\ 0 & \text{else} \end{cases} \tag{3.51}$$

with γ_{vi}, γ_r and γ_σ being the valuation elasticities of the trust, return and standard deviation components, respectively. Because the expected return may also be negative $\gamma_r \in \mathbb{N}^{+,\mathrm{odd}}$ must be a positive and odd natural number. Note that by dividing by their respective maximums all components of the Cobb-Douglas function can be maximally one. Furthermore, we introduce a cut-off $U^{\min}$ in order to avoid that investment bank agents provide very small amounts of interbank loans to commercial bank agents with a low valuation. Once commercial banks have been evaluated, a discrete choice model is employed in order to compute the exact allocation of funds from agent i to agent c:

$$a^I_{i,c,t} = \frac{\exp\left(g^A \frac{U^I_{i,c,t}}{\max(U^I_{i,c,t})}\right)}{\sum_{c=1}^{n^C} \exp\left(g^A \frac{U^I_{i,c,t}}{\max(U^I_{i,c,t})}\right)}, \tag{3.52}$$

with $g^A \geq 0$ being a parameter determining how strongly investment bank agents discriminate between the valuations of different commercial bank agents. When $g^A = 0$ interbank loans are distributed equally to commercial banks regardless of their valuation, while $g^A = \infty$ implies that only the agent with the highest valuation will be offered interbank loans.

By multiplying the allocation ratio with the composite interbank asset weight, we obtain the demand for short term interbank loans, i.e. $I_{i,c,t} = a^I_{i,t} a^I_{i,c,t} E_{i,t}$. Since investment banks compete in the market for interbank loans, but are unable to observe the conditions of competing offers, they will pay attention to the discrepancy $\Delta I_{i,c,t}$ between the supply and demand for interbank loans.

$$\Delta I_{i,c,t} = \begin{cases} I_{c,i,t} - I_{i,c,t} & \text{if } I_{i,c,t} \geq I_{c,i,t} \\ \frac{I_{c,CB,t} U^C_{c,i,t}}{\sum_{i=1}^{n^I} U^C_{c,i,t}} & \text{if } I_{i,c,t} < I_{c,i,t} \end{cases} \tag{3.53}$$

Note that while investment bank agent i can observe the excess supply of interbank loans, we assume that it perceives the excess demand of commercial bank agent c proportionally to the relative attractiveness $U^C_{c,i,t} / \sum U^C_{c,i,t}$ of its offer.

Offers made by an investment bank agent on the interbank market are binding, which implies that excess supply of credit leads to non-interest-bearing cash holdings, while excess demand suggests a foregone profit. It is therefore in the interest of investment bank agents to minimize $\Delta I_{i,c,t}$ by negotiating with commercial bank agents over the volume and interest rate of short term interbank debt. We model these negotiations via an iterative algorithm that in essence performs the task of a Walrasian auctioneer: The auctioneer posts an interest rate, checks loan supply and demand at the posted rate, reacts to the discrepancy between supply and demand by adjusting the interest via a logarithmic impact function and repeats the procedure.[16] For the sake of confining computational complexity, the auction is terminated prematurely. Instead of continuing the auction until demand exactly matches supply, it is terminated after Φ_t iterations, after which all transactions on the interbank market will take place. The logarithmic impact function used to adjust interest rates has the following form:

$$\log(r^I_{i,c,t+\frac{\phi}{\Phi_t}}) = \log(r^I_{i,c,t+\frac{\phi-1}{\Phi_t}}) + g^I \frac{\Delta I_{i,c,t+\frac{\phi-1}{\Phi_t}}}{|I_{c,i,t+\frac{\phi-1}{\Phi_t}}| + |I_{i,c,t+\frac{\phi-1}{\Phi_t}}|}, \tag{3.54}$$

with $\phi \in \{1, 2, ..., \Phi_t\}$ being the iteration count and $g^I > 0$ being a parameter determining the intensity with which interest rate adjustments take place in dependence of the differences in demand and supply on the interbank market. By dividing $\Delta I_{i,c,t}$ by the sum of absolute values of loan demand and loan supply, we bound changes to the interest rate within one iteration. The maximum percentage change of the interest rate in either direction is approximately the value of the intensity parameter g^I.

3.2.3 Expectation formation

Investment bank agents need to form expectations about return and variance of nb-securities, bank bonds and short term interbank loans. The expected return of nb-security s is deduced

[16] During the iterative procedure, demand and supply for interbank loans are calculated with the same Equations that determine the final demand and supply, but taking into account the currently negotiated interest rate. Also, commercial bank agents act on the assumption that they can fully raise the desired funds from the investment bank agents. In the auctions, commercial bank agents therefore ignore the cost of central bank funding in their calculation of the target ratio of long term wholesale debt, i.e. $\mathrm{E}_{c,t+\frac{\phi}{\Phi}}[a^{CB}] = 0$ and therefore $\mathrm{E}_{c,t+\frac{\phi}{\Phi}}[\bar{r}^I] = \mathrm{E}_{c,t+\frac{\phi}{\Phi}}[r^I]$.

by forecasting its default probability Ω_s^S and its price P_s^S. Taking into account the exogenously given maturity m_S and nominal interest rate $\bar{r}_{s,t}^S$, the expected return of asset s is computed as follows:

$$\underset{i,t}{\mathrm{E}}[r_s^S] = (1 - \underset{i,t}{\mathrm{E}}[\Omega_s^S]) \left(\frac{V_s^S}{P_{s,t}^S Q_s^S} (\bar{r}_{s,t}^S + 1 - m_S) + \frac{m_S \, \mathrm{E}_{i,t}[P_s^S]}{P_{s,t}^S} - 1 \right) - \underset{i,t}{\mathrm{E}}[\Omega_s^S], \tag{3.55}$$

with $V_{s,t}^S$ being the nominal value of the nb-security and Q_s^S the number of outstanding shares. For the sake of simplicity, we assume that investment bank agents believe that prices adjust immediately so that they incorporate all available information (i.e. $\mathrm{E}_{i,t}[P_s^S] = P_{s,t}^S$) and that the loss in case of a default is expected to be 100%. An investment bank agent's assessment of the true default probability $\Omega_{s,t}^S$ is updated by evaluating fundamental news shocks $\Delta\Omega_{s,t}^S = \log(\Omega_{s,t}^S) - \log(\Omega_{s,t-1}^S)$ and by identifying and correcting past valuation errors:

$$\underset{i,t}{\mathrm{E}}\left[\log(\Omega_s^S)\right] = \underbrace{\underset{i,t-1}{\mathrm{E}}\left[\log(\Omega_s^S)\right]}_{\text{past valuation}} + \underbrace{(\Delta\Omega_{s,t}^S + \epsilon_{i,s,t}^S)}_{\text{evaluation of news}} + \underbrace{\theta^S \left(\log(\Omega_{s,t}^S) - \underset{i,t-1}{\mathrm{E}}\left[\log(\Omega_s^S)\right] \right)}_{\text{past error correction}} \tag{3.56}$$

The stochastic error term $\epsilon_{i,s,t}^S$ and the slow correction of past errors (i.e. $\theta^S < 1$) cause forecasts of the true default probability to differ. Disagreement about the true value of an asset is the necessary condition for the emergence of a functioning asset market.

The estimated variance of an nb-security that is incorporated into the portfolio optimization is computed from the risk that arises due to the expected probability of default and the exponentially weighted moving average of an agent's quadratic forecast error:

$$\begin{aligned} \underset{i,t}{\mathrm{Var}}(r_s^S) = &\left(1 - \underset{i,t}{\mathrm{E}}[\Omega_s^S]\right) \hat{\mathrm{E}}_{i,t} \left[\left(\frac{V_{s,t}^S}{P_{s,t}^S Q_{s,t}^S} (\bar{r}_{c,t-1}^S + 1 - m_S) + \frac{m_S P_{s,t}^S}{P_{s,t-1}^S} - 1 - \underset{i,t-1}{\mathrm{E}}[r_s^S] \right)^2, \psi^S \right] \\ &+ \underset{i,t}{\mathrm{E}}[\Omega_s^S] \left(-1 - \underset{i,t}{\mathrm{E}}[r_s^S] \right)^2 \end{aligned} \tag{3.57}$$

The higher the historical discrepancy between agent i's return expectation and the realized return, the higher will be the agent's perception of risk (variance). Estimates of covariances, on the other hand, account for historical co-movements in assets and are important for the purpose of building a diversified portfolio. We assume that all agents have the same estimates of the covariance $\hat{\mathrm{Cov}}_t(r_{s1}^S, r_{s2}^S, \psi^S)$ between two assets, which is computed as an exponentially weighted moving average from daily returns.

The expected return and variance of a short term interbank loan and a bond depend on the expected default probability of the corresponding commercial bank agent. Taking into account the exogenous loan default process $\rho_{c,t}^L$, we can approximate the default probability of commercial bank agents:

$$\Omega_{c,t}^C \approx \Pr\{\rho_{c,t}^L L_{c,t} \geq E_{c,t}\} = F_{c,t}^{\rho^L} \left(\frac{E_{c,t}}{L_{c,t}} \right), \tag{3.58}$$

with $F_{c,t}^{\rho^L}(\cdot)$ defining the cumulative distribution function of the loan default rate. Since data on equity $E_{c,t}$, loan volume $L_{c,t}$ and the loan default rate distribution of commercial bank agents is not readily available to investment banks on a daily basis, we model expectations of the default probability analogous to that of nb-securities:

$$\underset{i,t}{\mathrm{E}}[\log(\Omega_c^C)] = \underset{i,t-1}{\mathrm{E}}[\log(\Omega_c^C)] + \left(\Delta\Omega_{c,t}^C + \epsilon_{i,c,t}^\Omega\right) + \theta^\Omega \left(\log(\Omega_{c,t}^C) - \underset{i,t-1}{\mathrm{E}}[\log(\Omega_c^C)] \right), \tag{3.59}$$

with $\Delta\Omega^C_{c,t} = \log(\Omega^C_{c,t}) - \log(\Omega^C_{c,t-1})$ being the news shock, $\epsilon^{\Omega}_{i,c,t}$ denoting the stochastic valuation error and θ^{Ω} being the speed with which past valuation errors are corrected.

The derivation of expected return and variance of short term interbank loans and bank bonds is similar to that of nb-securities:

$$\underset{i,t}{\mathrm{E}}[r^I_c] = (1 - \underset{i,t}{\mathrm{E}}[\Omega^C_c]) r^I_{i,c,t} - \underset{i,t}{\mathrm{E}}[\Omega^C_c] \quad \text{and} \tag{3.60}$$

$$\underset{i,t}{\mathrm{E}}[r^B_c] = (1 - \underset{i,t}{\mathrm{E}}[\Omega^C_c]) \left(\frac{B_{c,t}}{P^B_{c,t} Q^B_{c,t}} (\bar{r}^B_{c,t} + 1 - m_B) + \frac{m_B \mathrm{E}_{i,t}[P^B_c]}{P^B_{c,t}} - 1 \right) - \underset{i,t}{\mathrm{E}}[\Omega^C_c]. \tag{3.61}$$

Note that because short term interbank loans mature overnight and therefore cannot be traded, they lack a price. As with nb-securities, we assume that investment bank agents expect tomorrow's bond price to equal the current price, i.e. $\mathrm{E}_{i,t}[P^B_c] = P^B_{c,t}$. The variance components of interbank loans and bonds are defined as follows:

$$\underset{i,t}{\mathrm{Var}}(r^I_c) = \left(1 - \underset{i,t}{\mathrm{E}}[\Omega^C_c]\right) \left(r^I_{i,c,t} - \underset{i,t}{\mathrm{E}}[r^I_c]\right)^2 + \underset{i,t}{\mathrm{E}}[\Omega^C_c] \left(-1 - \underset{i,t}{\mathrm{E}}[r^I_c]\right)^2 \tag{3.62}$$

$$\begin{aligned}\underset{i,t}{\mathrm{Var}}(r^B_c) = \left(1 - \underset{i,t}{\mathrm{E}}[\Omega^C_c]\right) \hat{\mathrm{E}}_{i,t} &\left[\left(\frac{B_{c,t}}{P^B_{c,t} Q^B_{c,t}} (\bar{r}^B_{c,t-1} + 1 - m_B) + \frac{m_B P^B_{c,t}}{P^B_{c,t-1}} - 1 - \underset{i,t-1}{\mathrm{E}}[r^B_c] \right)^2, \psi^B \right] \\ &+ \underset{i,t}{\mathrm{E}}[\Omega^C_c] \left(-1 - \underset{i,t}{\mathrm{E}}[r^B_c]\right)^2 \end{aligned} \tag{3.63}$$

With the individual expected return and variance components for short term interbank loans, we can compute the return and variance for the composite short term interbank asset:

$$\underset{i,t}{\mathrm{E}}[r^I] = \sum_{c=1}^{n^C} a^I_{i,c,t} \underset{i,t}{\mathrm{E}}[r^I_c] \tag{3.64}$$

$$\underset{i,t}{\mathrm{Var}}(r^I) = \sum_{c=1}^{n^C} (a^I_{i,c,t})^2 \underset{i,t}{\mathrm{Var}}(r^I_c) \tag{3.65}$$

The covariance $\hat{\mathrm{Cov}}_{i,t}(r^B_c, r^I, \psi^S)$ between the composite short term inberbank asset and bonds, between nb-securities and the composite asset $\hat{\mathrm{Cov}}_{i,t}(r^I, r^S_s, \psi^S)$ as well as between nb-securities and bonds $\hat{\mathrm{Cov}}_{i,t}(r^B_c, r^S_s, \psi^S)$ are computed as exponentially weighted moving averages of observed returns.

3.3 Exogenous agents

Several exogenous agents help to close the model. These are a central counterparty for repo transactions and short selling, an institutional investor, which provides deposits for investment bank agents, a market maker, which sets the prices and interest rates as well as a lender of last resort. Furthermore, we incorporate an agent labeled "rest-of-world", which trades on the same asset markets as investment banks do.

3.3.1 Rest-of-world agent

The rest-of-world agent (row-agent) is included into the setup in order to keep the simulated financial system from becoming artificially fragile and in order to introduce the concept of asset liquidity. In reality, when the usual buyers of specific assets (investment bank agents in our context) are constrained, it falls to outside investors to absorb the excess supply of those assets. The row-agent agent represents these outside investors, which could include e.g. pension funds, insurance companies, unregulated financial institutions or individual investors. Because the row-agent by assumption is not specialized in assessing and trading the assets in question, we assume that it demands a higher return. This implies that the price of assets must drop below the mean valuation of investment bank agents before the row-agent becomes active. The more reluctant the row-agent is to buying an asset of a given risk/return profile, the less liquid that asset will be.[17]

The demand of the row-agent for nb-securities and bank bonds is derived from a portfolio optimization problem similar to that of investment bank agents:

$$\mathbf{a}^*_{row,t} = \arg\max_{\mathbf{a}} \mathbf{a}'\mathrm{E}_{row,t}[\mathbf{r}] - 0.5\lambda_{row}\mathbf{a}'\mathbf{\Sigma}_{row,t}\mathbf{a} \qquad \text{s.t.} \tag{3.66}$$

$$\{a^S_{row,s,t}, a^B_{row,c,t}, a^C_{row,t}\} \geq 0 \quad \text{and} \quad \mathbf{a}'\mathbf{1} = 1, \tag{3.67}$$

with $\mathbf{a}_{row,t} = (a^S_{row,1,t}, ..., a^S_{row,n^S,t}, a^B_{row,1,t}, ..., a^B_{row,n^C,t}, a^C_{row,t})'$ being a weights vector defining the desired value of individual nb-securities and bank bonds as a multiple of equity $E_{row,t}$. The expected returns contained in the vector $\mathrm{E}_{row,t}[\mathbf{r}]$ and the estimated variances and covariances contained in the matrix $\mathbf{\Sigma}_{row,t}$ are computed analogously to those of investment bank agents as defined in Section 3.2.3. Expectations differ in two regards: first, because the row-agent represents a group of investors, its expectations of the default probabilities $\Omega^S_{s,t}$ and $\Omega^C_{c,t}$ are set to their respective true value. Second, we assume that the agent is willing to hold an asset to maturity and therefore ignores the price volatility of assets, i.e. price changes disappear from the variance estimates. In this context, it seems reasonable to assume that investors lacking the experience of trading a specific asset will not hold that asset in their trading book, which might be subject to mark-to-market accounting rules. They will rather identify the long term benefit from holding an asset that is undervalued to maturity. For the sake of simplicity, we assume that the row-agent cannot incur debt, which is implied by the constraint in Eq. (3.67). To ensure that enough funds are available to eventually absorb assets in a fire sale spiral, we model equity of the row-agent as a function of the difference between investment bank agents' current equity and equity target E^*:

$$E_{row,t} = \max\left\{ x^{row}\left(\sum_{i=1}^{n^I} E^*_i - E_{i,t}\right)^2, E^{\min}_{row}\right\}, \tag{3.68}$$

with $E^{\min}_{row}$ defining a fixed minimum equity of the row-agent and the parameter $x^{row} > 0$ setting the aggressiveness with which row-equity is increased when investment bank agents make losses.[18] The demand $\Delta Q^S_{row,s,t}$ for nb-security s can now be determined:

$$\Delta Q^S_{row,s,t} = \frac{a^S_{row,s,t}E_{row,t}}{P^S_{s,t}} - m^S Q^S_{row,s,t-1}, \tag{3.69}$$

[17] Our concept of the row-agent strongly relates to the seminal discussion on asset liquidity and debt capacity in Shleifer and Vishny (1992). It is also related to the empirical literature on price impacts in financial markets. See for example Coval and Stafford (2007) or Jotikasthira et al. (2012) for estimates of price impact in stock markets and Ellul et al. (2011) or Feldhuetter (2012) for estimates in bond markets.

[18] The function for $E_{row,t}$ is rather ad hoc. It is motivated by the notion that when asset prices fall below the respective values deemed fair by constrained investment bank agents, they will increasingly attract outside investors. A greater number of outside investors will have more equity and hence capacity to absorb assets.

with $Q^S_{row,s,t-1}$ denoting the quantity held by the row-agent in the last period. The demand for bank bonds is calculated in the same manner.

The vector λ_{row} contains a risk aversion parameter for each nb-security and bank bond. The parameters are set orders of magnitude larger than the risk aversion of investment bank agents. This ensures that the row-agent does not distort prices in normal times. By assigning an individual risk aversion to each asset in the portfolio of the row-agent, we can make some assets more liquid than others. The higher the specific risk aversion parameter of an asset is relative to other assets, the larger must be the price drop before the row-agent absorbs that asset. Fire sale dynamics of illiquid (higher risk aversion) assets therefore become more pronounced than for liquid (lower risk aversion) assets.

3.3.2 Central counterparty

We assume, for the sake of simplicity, that the central counterparty is always willing to engage in short selling and repo transactions with investment bank agents. The central counterparty thereby manages its risk by setting the haircut and margin requirement for the corresponding asset. The amount of cash the investor is willing to lend in a repo transaction depends on the current price of the collateral $P^S_{s,t}$ and the haircut $h_{s,t}$. Specifically, the central counterparty chooses the haircut so that the probability of the collateral being worth less than the loan provided does not exceed x^R, i.e. $\Pr\{P^S_{s,t+1} \leq P^S_{s,t}(1-h_{s,t})\} = \Pr\{r^S_{s,t} \leq -h_{s,t}\} = x^R$. With F_s^{-1} denoting the quantile function of the return of asset s in the next period, the haircut is computed as follows:

$$h_{s,t} = -F_s^{-1}(x^R). \tag{3.70}$$

The margin requirement $k_{s,t}$ can be computed analogously. Specifically, $k_{s,t} = F_s^{-1}(1-x^R)$, which is equal to $h_{s,t}$ if the probability density function of the return of the corresponding asset is symmetric around zero. As a compensation for its risk, the central counterparty will demand a small fee of r^R_t and r^M_t for repo and short selling transactions, respectively.

3.3.3 Investor deposits

Unlike customer deposits, investor deposits are uninsured and therefore potentially very volatile. The volume of funds investors are willing to lend depends on the profitability of the debtor agent and the speed at which the investor can withdraw funds. The maturity parameter m_D thereby ensures that deposits cannot be withdrawn at once, but at a constant rate of $1-m_D$ per period. The higher the maturity parameter, the slower funds can be withdrawn. Without specifying the return on deposits, we assume that it is proportional to the debtor agent's return on assets $\pi_{i,t}$. Since investors only loose money if the debtor agent defaults, it seems sensible that investors consider a default scenario when deciding on the volume of deposits. Specifically, investors consider a predefined stress scenario of a daily negative return on assets $\rho^D_{i,t}$, which eventually leads to the default of the debtor agent.[19] Under such a stress scenario the investor would try to withdraw its funds as quickly as possible. Taking this into account, the initial investment $D^*_{i,t}$ is chosen in such a way that the funds D^{risk}_i not withdrawn at the time of bankruptcy are smaller than a specified fraction $\alpha_D = D^{\text{risk}}_i / D^*_{i,t}$ of the initial investment. The law of motion for equity under the stress scenario is described by the following equation:

$$E_{i,t+\tau+1} = E_{i,t+\tau} + (E_{i,t+\tau} + D^*_{i,t} m_D^{\tau-1})\rho^D_{i,t} \tag{3.71}$$

[19] A stress scenario could e.g. be defined as a daily return on assets which is one standard deviation to the left of the expected return on assets, with the expected value and variance or the return on assets being historical estimates ($\rho^D_{i,t} = \hat{\mathrm{E}}_t[\pi_i, \psi^D] - \sqrt{\hat{\mathrm{Var}}_t(\pi_i, \psi^D)}$).

Note that, for the sake of simplicity, we assume that investor deposits are the only form of debt considered. From Eq. (3.71) it becomes clear that the higher the volume of investor deposits, the faster the debtor will default under the specified stress scenario. We derive the decision of the investor by determining first, how many periods T^{def} it takes until the fraction of initial deposits not yet withdrawn reaches α^D; and second, what the initial volume of deposits must be so that the debtor agent defaults after T^{def} periods under the stress scenario. Specifically, from the law of motion of investor deposits ($D_t = m_D D_{t-1}$) under the stress scenario (deposits are withdrawn as fast as possible), we compute $T^{\text{def}} = 1 + \log(\alpha_D)/\log(m_D)$. By setting equity from Equation (3.71) at period T^{def} to zero, we can derive (see Appendix D) the initial volume of deposits:

$$D^*_{i,t} = \frac{-E_{i,t}(1+\rho^D_{i,t})}{\rho^D_{i,t}\frac{1-\left(\frac{m_D}{1+\rho^D_{i,t}}\right)^{T^{\text{def}}}}{1-\frac{m_D}{1+\rho^D_{i,t}}}} \tag{3.72}$$

Under the setting described above, $D^*_{i,t}$ can be used to calculate the maximum leverage $\text{lev}^{max}_{i,t} = D^*_{i,t}/E_{i,t}$ of investment bank agent i at time t. Any bound on leverage can be translated into a haircut parameter. With $\text{lev}^{max}_{i,t} = 1/h^D_{i,t} - 1$ we can derive $h^{D^*}_{i,t} = E_{i,t}/(E_{i,t} + D^*_{i,t})$. Since investors cannot withdraw funds faster than they mature, the corresponding haircut has an upper bound, i.e. $h^D_{i,t} \leq E_{i,t-1}/(E_{i,t-1} + m_D D_{i,t-1})$. On the other hand, we assume that investment bank agents are able to buy back debt, i.e. reduce investor deposits regardless of their maturity.

3.3.4 Lender of last resort

The central bank acts as a lender of last resort by providing a marginal lending facility (discount window) to commercial bank agents. We assume that agents consider the central bank as a potential creditor in the interbank market.[20] It is ranked along with other creditors via Eq. (3.24) according to the same two factors as investment bank agents. Borrowing from the central bank comes at a price. The marginal lending rate r^{CB}_t is typically higher than interbank interest rates, which implies that $u_{c,CB,t} \ll 1$. Furthermore, we assume that commercial bank agents fear that making use of the marginal lending facility might tarnish their reputation.[21] To account for this the trust factor is set to its minimum, i.e. $v_{c,CB,t} = \Xi^{min}/\Xi^{max}$.

3.3.5 Market maker

Market prices for nb-securities and bonds in our model evolve endogenously according to demand and supply. In the literature, pricing mechanisms range from simple price impact functions, over market clearing prices to sophisticated order book models (see e.g. Farmer and Joshi, 2002; Arthur et al., 1996; Farmer et al., 2005, respectively). Since the derivation of demand for an asset from the portfolio optimization problem of investment bank agents requires the agents' knowledge of the prices at which they can trade, we choose to price assets via an exogenous market maker. We assume that the market maker lacks information about the fundamentals of nb-securities and bonds. In order to limit its exposure to the risky assets, the market maker tries to learn the prices at which demand and supply for the respective assets are balanced. In practice, learning about the true price is facilitated by the sequential process of trading and

[20] In practice, making use of the marginal lending facility requires a bank to post collateral in exchange for central bank money. However, non-marketable assets (including bank loans) are also eligible as collateral if they are rated above a certain threshold.

[21] This is consistent with the observation that even after the default of Lehman Brothers, only poorly performing US banks accessed the discount window (see Afonso et al., 2011).

constant adjustments of bid and ask prices. Since in the context of our model all trading occurs simultaneously, we choose the incomplete Walrasian auction introduced in Section 3.2.2 as the pricing mechanism.

In order to find the appropriate price of an nb-security, the market maker has to determine the market interest rate first. As in Eq. (3.54) the interest rate results from Φ_t iterations of a logarithmic impact function, which depends on the excess demand (normalized by the trading volume) of the market maker, the investement bank agents and the row-agent:

$$\log(r^S_{s,t+\frac{\phi}{\Phi_t}}) = \log(r^S_{s,t+\frac{\phi-1}{\Phi_t}}) + g^{MMS}\left(\frac{-Q^{MMS}_{s,t} + \sum_{i=1}^{n^I} \Delta Q^S_{i,s,t+\frac{\phi-1}{\Phi_t}} + \Delta Q^S_{row,s,t+\frac{\phi-1}{\Phi_t}}}{|Q^{MMS}_{s,t}| + \sum_{i=1}^{n^I}\left|\Delta Q^S_{i,s,t+\frac{\phi-1}{\Phi_t}}\right| + \left|\Delta Q^S_{row,s,t+\frac{\phi-1}{\Phi_t}}\right|}\right), \tag{3.73}$$

with $\phi \in \{1, 2, ..., \Phi_t\}$ being the iteration count, $g^{MMS} > 0$ being the intensity of interest rate adjustments and $Q^{MMS}_{s,t} = m_S Q^{MMS}_{s,t-1} + (1 - m_S)Q^S_s - \sum_{i=1}^{n^I} \Delta Q^S_{i,s,t} - \Delta Q^S_{row,s,t}$ denoting the market makers inventory. Note that the inventory of the market maker is increased by the maturing nb-securities ($(1 - m_S)Q^S_s$) each period. This implies that the total number of shares of nb-security s remains constant throughout the simulation. The market price of nb-securities can be calculated by employing a present value approach of the flow of interest payments and repayments discounted at the market interest rate $r^S_{s,t}$:

$$P^S_{c,t} = \sum_{\tau=1}^{\infty} \frac{V^S_{s,t}}{Q^S_s}(\bar{r}^S_{s,t} + 1 - m_S)\frac{m_S^{\tau-1}}{(1 + r^S_{s,t})^\tau} = \frac{V^S_{c,t}}{Q^S_s}\frac{\bar{r}^S_{s,t} + 1 - m_S}{r^S_{s,t} + 1 - m_S} \tag{3.74}$$

The interest rate and the price of bank bonds are determined analogously to those of nb-securities. The main difference is that the quantity of bonds is not constant but endogenously determined. Furthermore, we assume that the inventory of bank bonds of the market maker cannot be negative, i.e. short sales of bank bonds are not allowed. The bond interest rate is updated as follows:

$$\begin{aligned}\log(r^B_{c,t+\frac{\phi}{\Phi_t}}) =& \log(r^B_{c,t+\frac{\phi-1}{\Phi_t}}) \\ &- g^{MMB}\left(\frac{-Q^{MMB}_{c,t} - \frac{\Delta B_{c,t+\frac{\phi-1}{\Phi_t}}}{P^B_{c,t+\frac{\phi-1}{\Phi_t}}} + \sum_{i=1}^{n^I} \Delta Q^B_{i,c,t+\frac{\phi-1}{\Phi_t}} + \Delta Q^B_{row,c,t+\frac{\phi-1}{\Phi_t}}}{Q^{MMB}_{c,t} + \frac{\Delta B_{c,t+\frac{\phi-1}{\Phi_t}}}{P^B_{c,t+\frac{\phi-1}{\Phi_t}}} + \sum_{i=1}^{n^I}\left|\Delta Q^B_{i,c,t+\frac{\phi-1}{\Phi_t}}\right| + \left|\Delta Q^B_{row,c,t+\frac{\phi-1}{\Phi_t}}\right|}\right),\end{aligned} \tag{3.75}$$

with $g^{MMB} > 0$ being the impact factor and $Q^{MMB}_{c,t} = m_B Q^{MMB}_{c,t-1} + (\Delta B_{c,t}/P^B_{c,t}) - \sum_{i=1}^{n^I} \Delta Q^B_{i,c,t} + \Delta Q^B_{row,c,t}$ denoting the inventory of the market maker. $\Delta B_{c,t}$ is the additional value of long term loans the commercial bank agent c issued in period t and $P^B_{c,t}$ is the price for one unit of the newly issued loan derived in Eq. (3.76). The price of a bank bond is again derived from the present value approach:

$$P^B_{c,t} = \sum_{\tau=1}^{\infty} \frac{B_{c,t}}{Q^B_{c,t}}(\bar{r}^B_{c,t} + 1 - m_B)\frac{m_B^{\tau-1}}{(1 + r^B_{c,t})^\tau} = \frac{B_{c,t}}{Q^B_{c,t}}\frac{\bar{r}^B_{c,t} + 1 - m_B}{r^B_{c,t} + 1 - m_B} \tag{3.76}$$

In order to limit the number of iterations, but achieve a satisfactory balance of demand and supply for interbank short term loans, bonds and nb-securities, we define the following stopping criteria:

$$x^I \geq \operatorname*{median}_c \left(\frac{\left| I_{c,t+\frac{z^I-1}{z^B}} - \sum_{i=1}^{n^I} I_{i,c,t+\frac{z^I-1}{z^I}} \right|}{\max\left\{ I_{c,t+\frac{z^I-1}{z^I}}, \sum_{i=1}^{n^I} I_{i,c,t+\frac{z^I-1}{z^I}} \right\}} \right) \tag{3.77}$$

$$x^{MMS} \geq \frac{1}{n^S} \sum_{s=1}^{n^S} \left| \frac{-Q_{s,t}^{MMS} + \sum_{i=1}^{n^I} \Delta Q^S_{i,s,t+\frac{z^S-1}{z^S}} + \Delta Q^S_{row,s,t+\frac{z^S-1}{z^S}}}{|Q_{s,t}^{MMS}| + \sum_{i=1}^{n^I} \left| \Delta Q^S_{i,s,t+\frac{z^S-1}{z^S}} \right| + \left| \Delta Q^S_{row,s,t+\frac{z^S-1}{z^S}} \right|} \right| \tag{3.78}$$

$$x^{MMB} \geq \frac{1}{n^C} \sum_{c=1}^{n^C} \left| \frac{\sum_{i=1}^{n^I} m_B Q^B_{i,c,t} + \Delta Q^B_{i,c,t+\frac{z^B-1}{z^B}} + \Delta Q^B_{row,c,t+\frac{z^B-1}{z^B}}}{Q^B_{c,t} + \frac{\Delta B_{c,t+\frac{z^B-1}{z^B}}}{P^B_{c,t+\frac{z^B-1}{z^B}}}} \right| \tag{3.79}$$

with z^I, z^S and z^B defining the lowest iteration count for which the respective stopping criteria is fulfilled. In essence, a satisfactory balance between demand and supply of short term loans, nb-securities and bonds is achieved when the median or average of the relative discrepancy between supply and demand does not exceed x^I, x^{MMS} and x^{MMB}, respectively. Since all prices and interest rates contained in the portfolio optimization problem of investment bank agents need to be known simultaneously, the number of iterations Φ_t of the pricing mechanisms of short term and long term interest rates as well as nb-security prices, is the same. Specifically, $\Phi_t := \max\{z^I, z^S, z^B\}$.

4 Modelling the Liquidity Coverage Ratio

The liquidity coverage ratio (LCR) is part of the Basel III framework and has been designed in order to address the problem of insufficient liquidity in times of stress. Specifically, the regulation requires banks to hold sufficient high quality liquid assets (HQLA) in order to meet the expected net cash outflows over thirty days of stress. The stress scenario is thereby defined by the regulator via fixed run-off rates for liabilities, inflow rates for assets that do not count as HQLA and haircuts for assets that count as HQLA. The run-off rate and the inflow rate specify how much of the liabilities and assets cannot be rolled over in times of stress, whereas the haircut implies a potential loss in value of an HQLA in times of stress. The regulation that should be fully implemented by 2019 (2018 in the European Union) requires that in normal times the ratio of HQLA to net outflows is greater than or equal to one, i.e.

$$\text{LCR} = \frac{\text{HQLA}}{\text{net outflows of 30 days}} \geq 100\% \tag{4.1}$$

4.1 Commercial Banks

The only asset of a commercial bank agent that qualifies as a high quality liquid asset is cash. Under the LCR, commercial bank agents need to hold the following amount of cash:

$$
C_{c,t}^{LCR} = \begin{cases} 0.25 \cdot \text{outflows}_{c,t} & \text{if inflows} > 0.75 \text{ outflows} \\ \text{outflows}_{c,t} - \text{inflows}_{c,t} & \text{if inflows} \leq 0.75 \text{ outflows} \end{cases} \tag{4.2}
$$

Note that according to the LCR regulation, banks need to hold at least 25% of their outflows in HQLA. The outflows a commercial bank needs to consider result from potential withdrawals from customer deposits, bonds and short term interbank debt, which are quantified by the regulator through the run-off rates $w^D = 0.03$, $w^B = 1$ and $w^I = 1$, respectively:

$$
\begin{aligned}
\text{outflows}_{c,t} =& w^D D_{c,t} + w^I I_{c,t}(1 + \bar{r}^I_{c,t}) + w^B B_{c,t} \sum_{\tau=1}^{30} m_B^{\tau-1} (\bar{r}^B_{c,t} + 1 - m_B) \\
=& w^D D_{c,t} + w^I I_{c,t}(1 + \bar{r}^I_{c,t}) + w^B B_{c,t} \frac{1 - m_B^{30}}{1 - m_B} (\bar{r}^B_{c,t} + 1 - m_B) \\
=& w^D D_{c,t} + W_{c,t}(1 - a_{c,t}) \underbrace{w^I (1 + \bar{r}^I_{c,t})}_{c0_{c,t}} + \underbrace{w^B B_{c,t-1} m_B \frac{1 - m_B^{30}}{1 - m_B} (\bar{r}^B_{c,t-1} - r^B_{c,t})}_{c1_{c,t}} \\
& + W_{c,t} a_{c,t} \underbrace{w^B \left(\frac{1 - m_B^{30}}{1 - m_B} (r^B_{c,t} + 1 - m_B) \right)}_{c2_{c,t}}
\end{aligned} \tag{4.3}
$$

The inflows consider the expected interest payments and repayments of loans for which the regulator has determined an inflow rate of $w^L = 0.5$:

$$
\begin{aligned}
\text{inflows}_{c,t} &= w^L \sum_{\tau=1}^{30} m_L^{\tau-1} L_{c,t} (1 - \operatorname*{E}_{c,t}[\rho^L])^\tau (\tilde{r}^L_{c,t} + 1 - m_L) \\
&= L_{c,t} \underbrace{w^L (\tilde{r}^L_{c,t} + 1 - m_L)(1 - \operatorname*{E}_{c,t}[\rho^L]) \frac{1 - (m_L(1 - \mathrm{E}_{c,t}[\rho^L]))^{30}}{1 - m_L(1 - \mathrm{E}_{c,t}[\rho^L])}}_{c3_{c,t}},
\end{aligned} \tag{4.4}
$$

In Eq. (4.3), we have rewritten short term debt $I_{c,t} = W_{c,t}(1 - a_{c,t})$, long term debt $B_{c,t} = W_{c,t} a_{c,t}$ and average long term interest rates $\bar{r}^B_{c,t} = \frac{B_{c,t-1} m_B \bar{r}^B_{c,t-1} + (W_{c,t} a_{c,t} - B_{c,t-1} m_B) r^B_{c,t}}{W_{c,t} a_{c,t}}$ as a function of wholesale debt $W_{c,t}$ and the long term debt ratio $a_{c,t}$. It thereby becomes clear that outflows are a function of the maturity structure of wholesale debt. With the balance sheet identity from Eq. (3.8), we can now calculate the necessary volume of wholesale debt under the LCR regulation:

$$
\begin{aligned}
W_{c,t} &= L_{c,t} - E_{c,t} - D_{c,t} + C^{LCR}_{c,t} \\
&= \begin{cases} \frac{L_{c,t} - E_{c,t} - D_{c,t}(1 - 0.25 w_D) + 0.25 c1_{c,t}}{1 - 0.25((a_{c,t} \cdot c2_{c,t}) + (1 - a_{c,t}) c0_{c,t})} & \text{if inflows} > 0.75 \text{ outflows} \\ \frac{L_{c,t}(1 - c3_{c,t}) - E_{c,t} - D_{c,t}(1 - w_D) + c1_{c,t}}{1 - ((a_{c,t} \cdot c2_{c,t}) + (1 - a_{c,t}) c0_{c,t})} & \text{if inflows} \leq 0.75 \text{ outflows} \end{cases}
\end{aligned} \tag{4.5}
$$

Since under the LCR the necessary volume of wholesale debt is dependent on the composition of wholesale debt, the optimal value of $a_{c,t}$ is no longer given by Eq. (3.20). When including Eq. (4.5) into the calculation of the interest surplus of Eq. (3.19), the solution for the optimal maturity structure becomes more complex and needs to be solved numerically.

There are situations where it is sensible for a commercial bank agent not to comply with the LCR regulation. Technically, when the denominator of Eq. (4.5) approaches zero or drops below zero, the demand for wholesale debt becomes infinite. One unit of wholesale debt would require one or even more units of HQLA. In such cases commercial banks are allowed to hold less liquid assets than required by the LCR regulation. Furthermore, we allow agents to fall short of a LCR of 100% when they are unable to obtain sufficient wholesale funding and have to access the central bank's marginal lending facility. The implementations of the LCR regulation in different regions typically allow banks to be non-compliant under extraordinary circumstances. In such situations, a plan detailing how and when compliance with the LCR can be restored would need to be negotiated with the regulator.

4.2 Investment Banks

In our model nb-securities on investment bank agents' balance sheets qualify as high quality liquid assets for which a haircut w_s^S needs to be applied. The possibility to count nb-security s as a liquid asset, however, depends on whether the asset is used as collateral or not. Only if the asset is not used as collateral in a repo-transaction will it add to the stock of liquid asset. For a repo based on nb-security s, a run-off rate of w_s^R applies. The simplest way of complying with the LCR regulation entails choosing a funding structure for every investment decision that is LCR-neutral. With other words, the purchased asset itself satisfies the HQLA requirement after considering the expected outflows of its funding under stress. Given that an nb-security in our model can be financed by repos or investor deposits, the fraction $\alpha_{i,s,t}^R \in [0,1]$ of repo-financing of nb-security s can be computed as follows:

$$\underbrace{\alpha_{i,s,t}^R w_s^R (1-h_{s,t}^R) + (1-\alpha_{i,s,t}^R)(1-h_{i,t}^D) \sum_{\tau=1}^{30} m_D^{\tau-1}(r_{i,t}^D + 1 - m_D)}_{\text{expected outflow of one unit of nb-security } s \text{ under stress}} = \underbrace{(1-\alpha_{i,s,t}^R) w_s^S}_{\text{HQLA}}$$

$$\alpha_{i,s,t}^R = \frac{w_s^S - (1-h_{i,t}^D)c4_{i,t}}{w_s^R(1-h_{s,t}^R) + w_s^S - (1-h_{i,t}^D)c4_{i,t}}, \quad \text{with } \; c4_{i,t} = \frac{1-m_D^{30}}{1-m_D}(r_{i,t}^D + 1 - m_D) \tag{4.6}$$

Note that when an asset is used as collateral or adds to the stock of HQLA, no inflows from that asset are considered under the LCR regulation.

While nb-securities qualify as HQLA, short term interbank debt as well as bonds issued by commercial banks do not qualify as such. Funding investments in bank bonds and interbank loans therefore require complementary purchases of HQLA or an increase in cash holdings. This implies that the investor deposits needed to finance one unit of a bond from agent c will be $x_{i,c,t}^B$ times larger than the initial demand ($(1-h_{i,t}^D)a_{i,c,t}^B$) specified in Eq. (3.46). Considering the inflows and outflows connected to the purchase of a bond, the factor $x_{i,c,t}^B \geq 1$ is chosen in order to satisfy the following equation:

$$\underbrace{x_{i,c,t}^B a_{i,c,t}^B (1-h_{i,t}^D) c4_{i,t}}_{\text{outflow}} - \underbrace{a_{i,c,t}^B c5_{i,c,t}}_{\text{inflow}} = \underbrace{x_{i,c,t}^B a_{i,c,t}^B (1-h_{i,t}^D) - a_{i,c,t}^B (1-h_{i,t}^D)}_{\text{cash}}$$

$$x_{i,c,t}^B = \frac{c5_{i,c,t} - (1-h_{i,t}^D)}{(1-h_{i,t}^D)(c4_{i,t} - 1)}, \tag{4.7}$$

with

$$
\begin{aligned}
c5_{i,c,t} &= \sum_{\tau=1}^{30} (1 - \mathop{\mathrm{E}}_{i,t}[\Omega_c^C])^\tau m_B^{\tau-1} \frac{B_{c,t}}{P_{c,t}^B Q_{c,t}^B} (\bar{r}_{i,c,t}^B + 1 - m_B) \\
&= \frac{B_{c,t}}{P_{c,t}^B Q_{c,t}^B} (\bar{r}_{c,t}^B + 1 - m_B)(1 - \mathop{\mathrm{E}}_{i,t}[\Omega_c^C]) \frac{1 - (m_B(1 - \mathrm{E}_{i,t}[\Omega_c^C]))^{30}}{1 - m_B(1 - \mathrm{E}_{i,t}[\Omega_c^C])}
\end{aligned}
$$

As the LCR regulation requires banks to hold at least 25% of their outflows in HQLA, $x_{i,c,t}^B$ has a lower bound $x_{i,c,t}^{Bmin}$ that amounts to:

$$
\begin{aligned}
&0.25 x_{i,c,t}^{Bmin} a_{i,c,t}^B (1 - h_{i,t}^D) c4_{i,t} = x_{i,c,t}^{Bmin} a_{i,c,t}^B (1 - h_{i,t}^D) - a_{i,c,t}^B (1 - h_{i,t}^D) \\
&x_{i,c,t}^{Bmin} = \frac{1}{1 - 0.25 c4_{i,t}}
\end{aligned}
\tag{4.8}
$$

Analogously, we can calculate the factor $x_{i,t}^I \geq 1$ and its lower bound $x_{i,t}^{Imin}$ for the funding of the composite short term interbank loan:

$$
\begin{aligned}
&\underbrace{x_{i,t}^I a_{i,t}^I (1 - h_{i,t}^D) c4_{i,t}}_{\text{outflow}} - \underbrace{a_{i,t}^I \sum_{c=1}^{n^I} a_{i,c,t}^I c6_{i,c,t}}_{\text{inflow}} = \underbrace{x_{i,t}^I a_{i,t}^I (1 - h_{i,t}^D) - a_{i,t}^I (1 - h_{i,t}^D)}_{\text{cash}} \\
&x_{i,t}^I = \frac{\sum_{c=1}^{n^I} a_{i,c,t}^I c6_{i,c,t} - (1 - h_{i,t}^D)}{(1 - h_{i,t}^D)(c4_{i,t} - 1)} \geq x_{i,t}^{Imin} = \frac{1}{1 - 0.25 c4_{i,t}}, \\
&\text{with } \; c6_{i,c,t} = (1 - \mathop{\mathrm{E}}_{i,t}[\Omega_c^C])(1 + r_{i,c,t}^I).
\end{aligned}
\tag{4.9}
$$

In order to integrate the LCR regulation into the portfolio optimization problem of investment bank agents, we need to modify the constraints in Eq. (3.44) and (3.46) as follows:

$$
a_{i,s,t}^R = \begin{cases} -(1 - h_{s,t}^R) a_{i,s,t}^S \alpha_{i,s,t}^R & \text{if } \; a_{i,s,t}^S \geq 0 \\ 0 & \text{else} \end{cases}
\tag{4.10}
$$

$$
\begin{aligned}
a_{i,t}^D = &- (1 - h_{i,t}^D) \Bigg(a_{i,t}^I (1 + (1 - h_{i,t}^D)(x_{i,t}^I - 1)) + \sum_{c=1}^{n^C} a_{i,c,t}^B (1 + (1 - h_{i,t}^D)(x_{i,c,t}^B - 1)) + \\
&+ \sum_{s \in \mathcal{D}} a_{i,s,t}^S (1 - \alpha_{i,s,t}^R) \Bigg),
\end{aligned}
\tag{4.11}
$$

with $\mathcal{D} := \{s | a_{i,s,t}^S \geq 0\}$.

In general, the constraints make sure that investment decisions are LCR-neutral at all times. They enforce a funding mix that leads investment bank agents to be compliant with the LCR regulation. The optimal portfolio choice therefore becomes dependent on the run-off rates for repos, the haircuts for individual nb-securities and the maturity of investor deposits.

5 Simulations

The simulations presented in this paper are conducted with an uncalibrated model, which implies that only qualitative inferences are feasible in the current setup.[22] Nevertheless, the initialization of the model is not entirely arbitrary. While in the following we only comment on very few parameter choices, all parameters and initial values are reported in Tables 4 and 5 of the appendix. We simulate the model with $n^C = 100$ commercial bank agents, $n^I = 30$ investment bank agents and $n^S = 15$ nb-securities.[23] Nb-securities should not be interpreted as individual assets, but rather as large portfolios of assets. We assume that investment bank agents, although fewer in number, are larger than commercial bank agents in terms of balance sheet size (induced by a larger equity target). Customer deposits are kept rather low (5 times the equity target) in order to evoke a market for wholesale funding. This is consistent with the capital structure of large commercial banks in reality. The fluctuations in customer deposits are set to represent a rather calm economic environment. This also applies to the other two exogenous stochastic processes: the default rates for loans to the real sector and default probabilities of nb-securities. We thereby want to reproduce the seemingly stable financial system prior to the financial crisis. Each simulation run lasts for $T = 2000$ periods, whereby we discard the first 750 periods in order to reduce the impact of initial values. Each period represents a trading day and 250 periods a trading year.

The rest of this section is divided into two parts: A brief discussion of heterogeneity is followed by a more in-depth impact assessment of the liquidity coverage ratio regulation. Showing the emergent heterogeneity in size, leverage, agent defaults, asset returns and interest rates conveys an impression of the functioning of the model. A comparison between the emergent distributions and real data can furthermore serve as a first - although tentative - validation instrument. Our analysis of the liquidity coverage ratio regulation, will, on the other hand, focus on average effects rather than on distributional ramifications. The impact on balance sheets, on the maturity structure of wholesale debt, on monetary policy and stability will be assessed.

5.1 Heterogeneity and Distributions

Our model contains multiple sources of heterogeneity. The inclusion of two distinct bank business models is the most apparent source, but heterogeneity can also be imposed by setting parameters (e.g. equity targets, risk aversion, etc.) and initial values (e.g. initial balance sheet positions), or produced by endogenous dynamics (size, leverage, portfolio composition, etc.). While the high dimensionality of heterogeneity imposed through parameters and initial values will be useful when calibrating the model to match micro data, we will focus on endogenous sources of heterogeneity here.

Figure 2 shows the size and leverage distributions of commercial bank agents and investment bank agents after 2000 periods. To obtain more representative distributions, simulations are repeated 20 times with different initializations of the random number generator. The values for size and leverage of the different simulation runs are displayed jointly. The size distributions of the two agent types plotted in Figures 2(a) and 2(b) differ substantially. While the distribution for commercial bank agents is skewed to the right, the opposite is true for investment bank agents. The distributions thereby reflect the different business models of the agents. In essence, the size distribution of commercial bank agents reflects the default rate of loans to the real sector,

[22] In general, the extent of the model allows for a relatively accurate mapping of model parameters to micro data. The concepts and data needed for this endeavor will be the subject of a forthcoming paper.

[23] The number of agents and nb-securities seems low when compared to reality. However, taking into account, for example, that in the EU the largest 5% of banks (approximately 140 institutions) held about 90% of total banking assets in 2006, makes the choice appear more realistic.

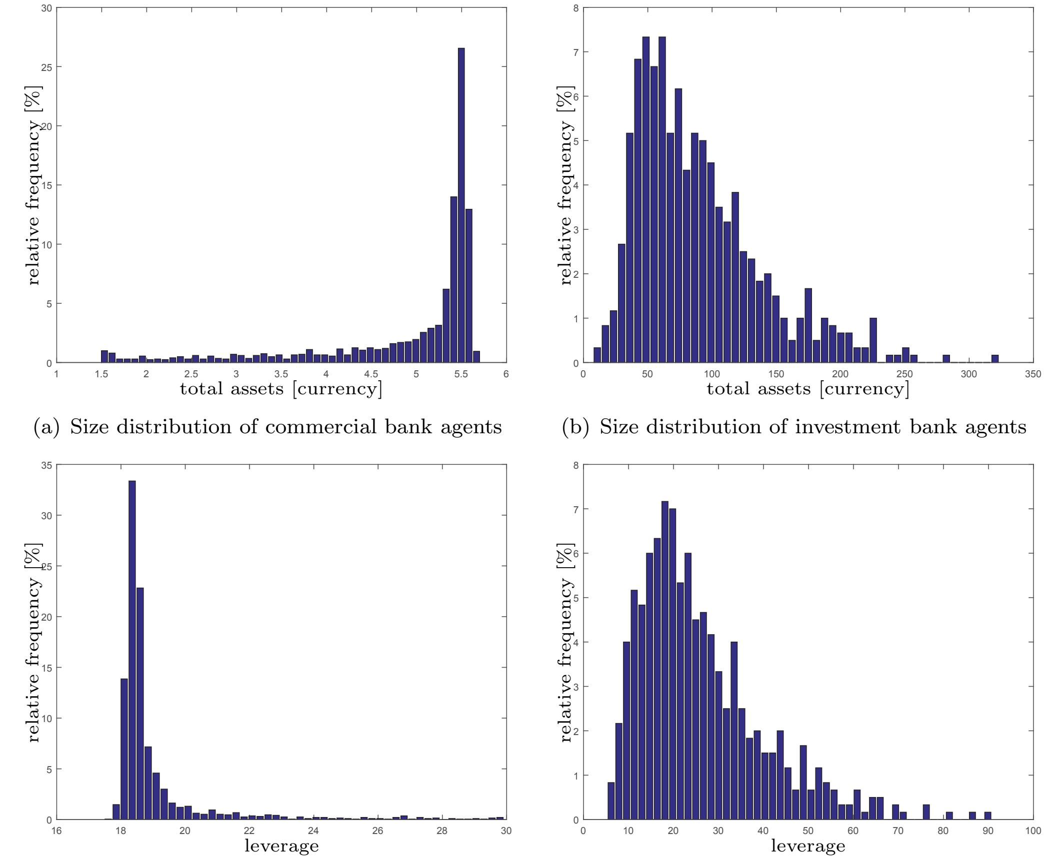
which we model with a log-normal distribution. Low default rates, which are the norm given their log-normal distribution, make most commercial bank agents rather successful in reaching their equity target. As in the current setup commercial bank agents do not differ regarding their equity targets, loan characteristics, expectations and risk management, a right skewed size distribution emerges naturally. More heterogeneity in the corresponding parameters will lead to a more pronounced right tail of the distribution. However, the fact that the size distribution of commercial banks in reality tends to be left-skewed, potentially indicates that some aspects of commercial banks' business models have been neglected in the model. Investment bank agents, on the other hand, do not supply loans, which remain on balance sheet until maturity, but trade large portfolios of assets. The emergent size distribution resembles a log-normal distribution, which is consistent with reality (see e.g. Fischer and Riedler, 2014). Invoking Gibrat's law, Fischer and Riedler (2014) have argued that the size of balance sheets converges to log-normality when agents primarily pursue trading activities.

The leverage distribution of commercial bank agents and investment bank agents is plotted in Figure 2(c) and 2(d), respectively. Interestingly, while size and leverage distributions of investment bank agents are rather similar, they seem mirror inverted for commercial bank agents. This suggests that small commercial bank agents tend to have high leverage ratios, while small investment bank agents have low leverage ratios on average. The reason for this can again be ascribed to the different business models of the agent types. The business model of investment bank agents allows them to expand and reduce their balance sheets quite quickly. When an agent expects that it can make a profit at low risk, it will take on more debt to buy the corresponding assets. When risk increases, the combination of short term funding and liquid assets allows the investment bank agent to reduce its leverage ratio by shrinking its balance sheet. Adrian and Shin (2010) have shown that this behavior is typical for investment banks in reality. On the other hand, commercial bank agents in our model hold illiquid assets which they, at least partially, finance with long term loans. When losses are incurred, the inability of a commercial bank agent to deleverage immediately, will lead to a temporary surge in the leverage ratio. With other words, while the relation between size and leverage of commercial bank agents is neutral most of the time, it becomes negative when the agents are shocked.

Whenever a leveraged agent incurs a loss, its equity can fall below zero and bankrupt the agent. Figure 3 plots the number of defaulting commercial bank agents per simulation run. In order to obtain the distribution, we simulate 1000 times and count all default events that occur each run between periods 750 and 2000 (5 trading years). The distribution is bell-shaped with a mean of approximately 11 defaults in 5 years. While default events are common within a simulation run, there is not one systemic event (a break down of the financial system) recorded in 1000 simulation runs. There are two main reasons for this. First, exogenous stochastic variables such as the default rates for loans to the real sector or default probabilities for nb-securities do not exhibit large macro shocks. The default rate of a commercial bank agent's loan portfolio may be high, but it is uncorrelated with the default rates of other agents. Second, unlike in many agent-based models of financial markets, agents in our model do not form expectations of prices or default probabilities by extrapolating past price movements.[24] Extrapolative expectations, which can lead to the emergence of bubbles and subsequent crashes, are known to be a source of endogenous instability. Their omission implies that our current setup is better suited for an analysis of average effects rather than extreme effects.

A key motivation for including different expectation formation processes into agent-based models of financial markets has been to explain the stylized facts of financial asset return time

[24]See e.g. chartist-fundamentalist approaches to modeling financial markets such as Lux and Marchesi (2000); Farmer and Joshi (2002); Westerhoff and Dieci (2006).



(a) Size distribution of commercial bank agents

(b) Size distribution of investment bank agents

(c) Distribution of leverage for commercial bank agents

(d) Distribution of leverage for investment bank agents

Figure 2: Size and leverage distribution of bank agents.

series.[25] Interestingly, our model does quite well at replicating these stylized facts although all investment bank agents form expectations in the same manner (see Eq. 3.55). Figure 4(a) plots the distribution of returns of an nb-security as well as a fitted Gaussian distribution. Clearly, the measured return distribution is leptokurtic, i.e. has a higher probability mass in the tails than the corresponding Gaussian probability density function. Figure 4(b) displays the autocorrelation structure of both raw returns as well as the absolute value of returns. The autocorrelation of the absolute values of returns is slowly decaying while raw returns are autocorrelated only for a short period of time. The initial significant positive autocorrelation in raw returns is an artifact of the pricing mechanism in our model. It allows for some degree of mis-pricing for the sake of reducing computational complexity. Fat tails and clustered volatility are the result of heterogeneous expectations. The degree to which agents disagree about the true value of an asset (disagreement is modeled in Eq. 3.56) determines the difference in the portfolios agents hold. Strong disagreement will decrease the portfolio overlap between agents and vice versa. In general, when an agent is optimistic about the prospects of an asset, it will hold a relatively

[25] Financial return time series typically display fat tails (excess kurtosis), clustered volatility (persistence in the amplitude of returns) and the absence of autocorrelation in raw returns (see e.g. Cont, 2001).

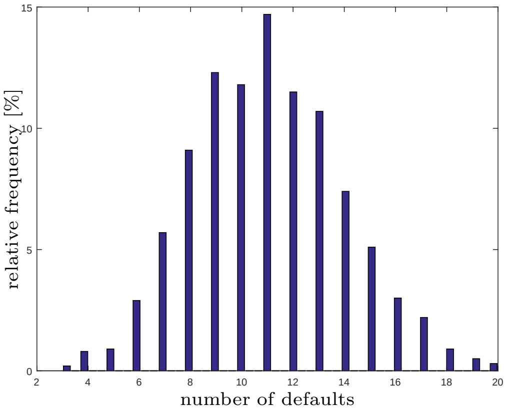
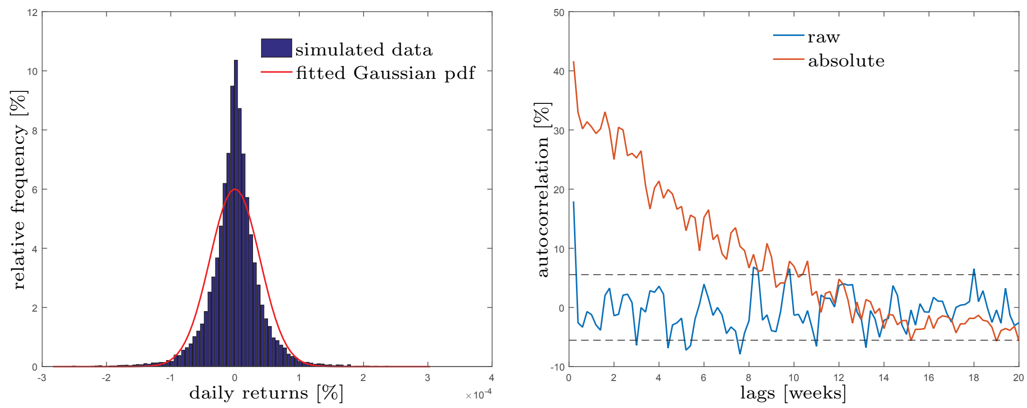


Figure 3: Distribution of defaulting commercial bank agents per simulation run (5 years).

large share of that asset. Most of the time, a gradual updating of expectations will lead to trade with a limited impact on prices. However, when an agent which is optimistic about the value of a specific asset incurs unexpected losses (e.g. when a commercial bank debtor defaults), it will start selling that asset into a market which maintains a more pessimistic valuation. The higher the disagreement, the higher will be the price impact and hence excess kurtosis of returns. In an interconnected system, the price drop of one asset also affects other assets. Further losses are incurred to investment bank agents. As their capacity to hold assets declines, the rest of the world agent (row-agent) steps in. The higher risk aversion of the row-agent[26] increases the volatility of the affected assets. Until investment bank agents recover and absorb the assets held by the row-agent, volatility will remain elevated. As a result, higher disagreement also translates into higher persistence of return amplitudes (volatility clustering).



(a) Return distribution

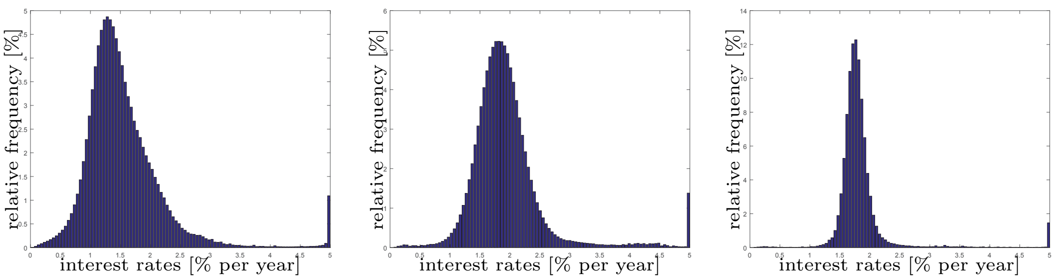
(b) Autocorrelation of returns

Figure 4: Return distribution and autocorrelation of returns of nb-security.

Emergent heterogeneity can also be observed in short term and long term wholesale debt

[26]The row-agent represents institutions and individual investors who lack the expertise to properly assess the value of nb-securities. This lack of expertise makes them reluctant to hold these assets at the price investment bank agents consider fair.

interest rate distributions. Fundamentals, such as equity and risk as well as deviating assessments of fundamentals, lead to differences in wholesale interest rates between commercial bank agents. Because interest rates themselves influence the dynamics of balance sheet variables (e.g. equity through lower or higher profit rates) they are both, a result and a source of heterogeneity. Figure 5 plots the emergent distributions of short term interbank rates, bank bond interest rates and the wholesale funding cost. The short term rates represent the weighted averages of interest paid on the interbank market, which includes agents' access to the marginal lending facility. The upper bound to short term interest rates is 5% per year, the rate at which commercial bank agents can borrow from the central bank. In the current setup, the median (over all agents, time and simulation runs) short term interest rate amounts to 1.42 % per year, which is 43 basis points lower than the median interest rate of bank bonds (1.85% per year) with an average maturity of 10 months ($m_B = 0.995$). Unlike short term interest rates, bank bond rates have no upper limit. When a commercial bank agent is close to default, interest rates on outstanding bank bonds will soar. To allow a better visual comparison of distributions, all bond interest rates higher than 5% per year are pooled at 5%. This concerns about 1.5% of the total sample of bank bond rates, with the maximum interest rate amounting to over 300% per year. The third distribution in Figure 5 shows the actual wholesale funding costs paid by commercial bank agents. With 1.76% per year, median wholesale fundings costs lie between the median short term rate and the median interest rate for bank bonds. Furthermore, the standard deviation of actual wholesale fundings costs is notably lower than the standard deviation of its components. The reason for this is that commercial bank agents try to minimize their funding costs by choosing a suitable funding structure and preferably borrowing when interest rates are low.



(a) short term interbank rates (b) bond interest rates (c) wholesale funding costs

Figure 5: Interest rate distributions of wholesale debt.

5.2 Assessing the impact of the LCR regulation

In order to assess the impact of the liquidity coverage ratio (LCR) regulation, we compare the results of simulations under two different setups: the benchmark setup without the liquidity coverage ratio and the LCR setup that includes the extensions of the model introduced in Section 4. Simulations within each setup are repeated 20 times. While the random seeds for the stochastic elements differ for the 20 simulation runs within one setup, they are identical across setups. This facilitates the comparison of both setups and at the same time reduces the probability that the documented results are due to chance.

5.2.1 Impact on Balance Sheets

In Table 1 we report the ratios of balance sheet positions to total assets for commercial bank agents with and without the LCR. The ratios document median values across time, agents and simulation runs. In the column labeled "change", we report the difference between the balance sheet positions under the two setups divided by the volume of total assets under the benchmark setup. This allows for a comparison of levels in addition to a comparison of ratios. Beside the level of customer deposits, which is exogenous to the model, all differences in levels are highly significant. The standard deviations, reported in parenthesis, describe the variation of the respective balance sheet ratios across simulation runs.

Table 1: Median balance sheet of a commercial bank agent with and without the liquidity coverage ratio (LCR). The number in parentheses is the standard deviation in percent of total assets.

	without LCR [% total assets]	with LCR [% total assets]	change [% to benchmark]
loans	99.52	96.99	-2.14
	(0.06)	(0.21)	
cash	0.48	3.01	+540.71
	(0.06)	(0.21)	
equity	5.23	5.17	-0.75
	(0.02)	(0.03)	
deposits	30.46	30.33	0.00
	(0.37)	(0.44)	
short term interbank	8.05	0.45	-94.41
	(0.84)	(0.13)	
bonds	56.27	63.83	+13.92
	(1.09)	(0.56)	
total assets	100.00	100.00	+0.41

The most striking impact of the LCR on balance sheet ratios is unsurprisingly the change in the cash position, which represents the high quality liquid asset (HQLA) within our framework. While commercial bank agents try to avoid cash holdings in the benchmark setup, they are obliged to hold cash when complying with the LCR regulation. Although the relative change in cash holdings is very large (over 540%), the median cash to total asset ratio under the LCR setup is only slightly higher than three percent.[27] Interestingly, the increase in HQLA (cash) leads to a 2.14% decrease in the level of loans the median commercial bank provides to the real sector.[28] This is not a trivial result, since the observed substitution effect could have been avoided by a sufficiently large expansion of agents' balance sheets. The observed increase in balance sheet size (+0.41%) is not enough to avoid a reduction in loans when compared to the benchmark setup. Balance sheets can be expanded by taking on more debt (i.e. a lower equity to total assets ratio), which does not necessarily mean that risk increases under the LCR setup. Since cash cannot loose value in our model, expanding the asset side with cash does not add any risk.

[27] In reality, there may be several reasons for commercial banks to hold liquid assets even without any liquidity regulation. The provision of credit lines to households or firms, for example, would be a good reason to hold a stock of HQLA on the balance sheet. Also, meeting reserve requirements will lead banks to hold HQLA.

[28] Impact assessments that rely on statistical relationships in historical data also suggest that the LCR regulation will tend to decrease loan supply. Estimated impacts range from 3-5% (see Figure 3-7 in Office of Financial Research, 2014).

On the liabilities side, however, the refinancing risk may increase when the expansion is financed with wholesale debt, in particular short term wholesale debt. The negative change in the level of equity (-0.75%) suggests that commercial bank agents have become less profitable under the LCR regulation. This is due to higher wholesale funding costs, which are induced by a drastic change in the funding structure of commercial bank agents under the LCR setup. The short term interbank market almost completely breaks down and overnight debt is replaced by longer term bank bonds, which increase by 13.92%. When the LCR is binding, it becomes impossible to finance long term illiquid assets with overnight debt. Since buying HQLA with overnight debt is not part of commercial bank agents' business models, the interbank market breaks down. The increased demand for long term wholesale funding leads to increasing funding costs (interest rates of bank bonds increase by about 15 basis points) as documented in Table 3. In general, we would assume that the LCR will shift demand towards funding sources with higher maturity and thereby steepen the term structure of uninsured wholesale debt.[29]

Table 2: Median balance sheet of an investment bank agent with and without the liquidity coverage ratio (LCR). The number in parentheses is the standard deviation in percent of total assets.

	without LCR [% total assets]	with LCR [% total assets]	change [% to benchmark]
non-bank-securities	84.40	83.21	+0.12
	(0.38)	(0.43)	
cash	3.50	4.71	+36.77
	(0.18)	(0.19)	
bank bonds	10.20	11.46	+14.10
	(0.27)	(0.26)	
interbank loans	3.96	0.00	-99.99
	(0.14)	(0.00)	
margin account	0.60	0.61	+4.85
	(0.05)	(0.04)	
equity	3.96	3.96	+1.56
	(0.17)	(0.15)	
investor deposits	11.05	23.32	+114.35
	(0.19)	(0.19)	
repos	84.39	72.10	-13.24
	(0.38)	(0.36)	
short sales	0.60	0.61	+4.85
	(0.05)	(0.04)	
total assets	100.00	100.00	+1.55

Table 2 compares the balance sheet ratios under the benchmark setup and the LCR setup for the median investment bank agent.[30] Through the interaction with commercial bank agents,

[29] Bech and Keister (2013) derive a similar conclusion with a different method. They introduce the LCR regulation into a standard model of banks' reserve management and find that the short end of the yield curve tends to get steeper when banks are concerned about violating the LCR. Furthermore, statistical impact assessments of the LCR regulation, summarized in Office of Financial Research (2014), predict increases in interest rates between 15 and 30 basis points.

[30] Investment bank agents in our model can represent the investment banking arm of a commercial bank, but also an institution (e.g. a hedge fund, broker dealer, structured investment vehicle, etc.) that is part of the shadow banking system and therefore not subject to normal financial regulation. In the following, we assume that

investment bank agents decrease their short term interbank lending and increase their holdings of bank bonds. The higher interest rate on bank bonds increases the profitability of the median investment bank agent, which leads to a higher level of equity when compared to the benchmark case. While the higher ratio of cash to total assets reduces the risk of the asset side, the shifting from short term interbank loans to long term bank bonds increases risk. In sum, the unchanged equity to total assets ratio suggests that the reallocation of assets has been risk neutral. The increase in balance sheet size by 1.55%, which is due to the higher profitability of investment bank agents has also led to an increase in nb-security holdings. This leads to a decrease in the interest rate of nb-securities (see Table 3). Lower interest rates means higher prices, which increases the fraction of agents that deem a specific nb-security to be overvalued. As a result, the volume of short sales and correspondingly the volume of cash held in the margin account increase.

Table 3: Median interest rates by category with and without the liquidity coverage ratio (LCR). The number in parentheses is the standard deviation of interest rates across simulation runs in percent.

	without LCR [% per year]	with LCR [% per year]	change [% to benchmark]
non-bank-securities	0.035	0.033	-5.714
	(0.004)	(0.003)	
short term interbank	1.417	0.209	-81.779
	(0.023)	(0.012)	
bank bonds	1.806	1.948	+7.290
	(0.028)	(0.033)	

5.2.2 Impact on the Maturity of Wholesale Funding

The maturities of assets and liabilities are important inputs for calculating the liquidity coverage ratio. All else being equal, shorter maturities of loans to the real sector would increase the inflows and reduce the volume of HQLA required under the LCR regulation. Shorter maturities of wholesale debt, on the other hand, would increase outflows and lead to a higher demand for HQLA. It is therefore plausible that banks will consider changing the average maturity of their assets and/or liabilities when the LCR regulation is active. Unlike the choice between overnight interbank debt and long term wholesale debt (bank bonds), the average maturity m_B of bank bonds is exogenous to our model. In order to test whether commercial bank agents will be inclined to choose a different maturity structure for their bank bonds, we compare simulation results for different values of the maturity parameter m_B under the benchmark setup and the LCR setup. We assume that agents have an incentive to change their maturity structure if they can profit from such a change.

In Figure 6, we show the relation between the return on assets (RoA) of commercial bank agents and the average maturity[31] of their long term wholesale funding (bonds). The solid lines represent the median values over time, agents and simulation runs, while the shaded areas depict

the LCR applies for all investment bank agents. Qualitatively, most results remain valid when the LCR is only applied to commercial bank agents. An exception is the shift in the investment bank funding structure towards a higher ratio of investor deposits and a lower ratio of repo financing, which disappears when investment bank agents do not need to comply with the LCR regulation.

[31] The average maturity in months is computed with Eq. (3.5); we assume that a month has 20 trading days.

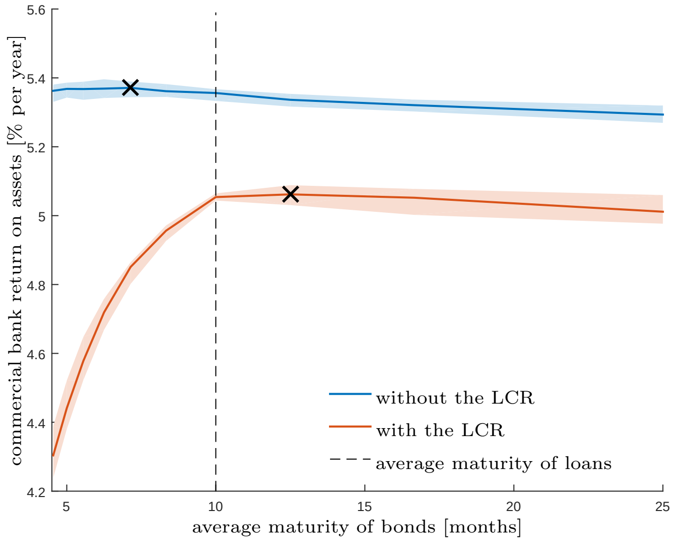


Figure 6: Commercial bank profit for different average maturities of bank bonds.

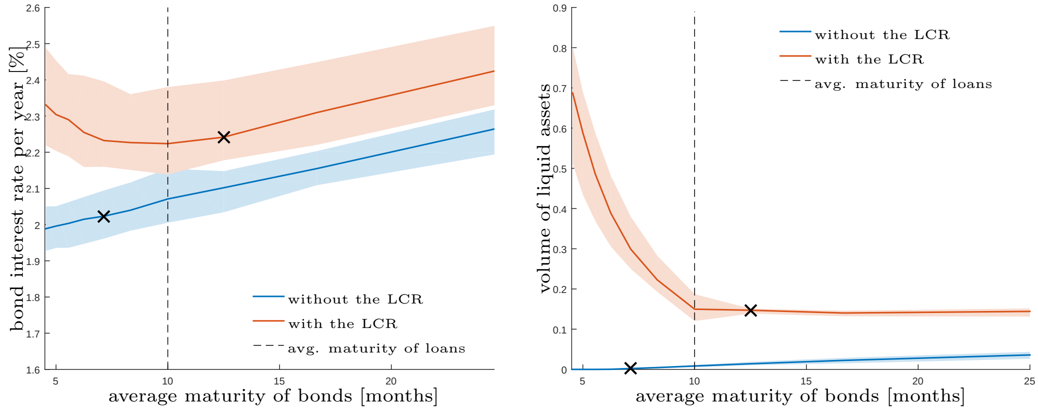
the variation (specifically, the 90% confidence interval) across simulation runs. The dashed line marks the average maturity of loans, which is kept constant, and the Xs highlight the average maturity for which the RoA is maximized. Figure 6 reveals three important findings: First, commercial bank agents are on average always more profitable under the benchmark setup than under the setup where the LCR is binding. Second, when the average maturity of bank bonds is greater than the average maturity of loans ($m_B > m_L$), the spread between the RoAs of the two setups remains more or less constant. The spread, however, widens dramatically when $m_B < m_L$. Third, the optimal average maturity of bank bonds under the benchmark setup is shorter than the average maturity of loans. The opposite is true under the LCR setup.

The third finding suggests that a commercial bank under the LCR regulation will have a considerable incentive to make sure that its wholesale funding has on average a greater maturity than the average maturity of its assets. This can be achieved either by increasing the maturity of bank bonds or decreasing the maturity of loans to the real sector. In both cases, banks would reduce the liquidity in the banking sector. Beside risk transformation, liquidity creation, which is achieved when banks' liabilities are more liquid than their assets, has been acknowledged as an important role of banks in the context of economic growth at least since Adam Smith (see Berger and Bouwman, 2009). Our model suggests that liquidity creation with wholesale debt will become more difficult when banks need to comply with the LCR regulation.[32]

The first and second findings are explained by Figure 7: As illustrated in the left panel, bond interest rates under the LCR setup are persistently higher than in the benchmark setup, which explains the lower profitability of commercial bank agents.[33] For $m_B \geq m_L$ bond interest rates

[32]Empirical evidence supports this model prediction. At least in the US, the funding structure of banks is undergoing changes in line with our simulation results. Wholesale funding is becoming increasingly longer term, while banks are increasing their volume of liquid assets and are providing fewer loans to the real sector (see e.g. Buehler et al., 2013). In Europe, on the other hand, wholesale funding is becoming more short term. However, the slow recovery after the financial and sovereign debt crises and ongoing financial fragility rather than liquidity regulation are arguably dominating balance sheet dynamics in the EU.

[33]Note that the specific result that interest rates are higher under the LCR and the general result that interest rates increase with increasing maturity are emergent phenomena of our model. These results are not explicitly written into the model equations, but emerge through the interactions of agents in the market. The lower



(a) bond maturity and bond interest rates

(b) bond maturity and cash holdings

Figure 7: Cash and bond interest rates for different maturities.

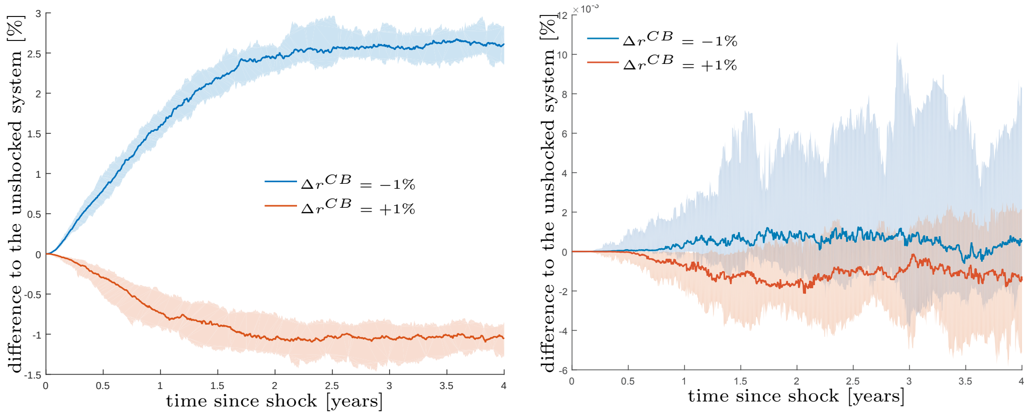
increase with increasing average maturity of bonds under both setups. The upward-sloping yield curve emerges due to risk considerations by investment bank agents. The longer the average maturity, the smaller will be the received repayment per period. When repayments are spread over a longer time horizon, it becomes more probable that the investor will suffer losses due to a default event or a reassessment of default probabilities. Investment bank agents seek compensation for this through higher interest rates. Expectations of lower or higher interest rates in the future do not play a role in our current setup. When commercial bank agents under the LCR setup choose an average maturity for their wholesale funding that is lower than the average maturity of loans, interest rates start increasing with declining m_B. A demand effect explains this result. As illustrated in the right panel of Figure 7 commercial bank agents need to hold an increasing volume of cash for very low average maturities of wholesale debt. Consequentially a higher volume of wholesale debt is needed to fund the same amount of loans to the real sector. This leads to higher interest rates. The increasing spread between the RoAs in Figure 6 for $m_B < m_L$ is thus partially explained by the higher interest rate for wholesale debt and partially by the higher share of low-yielding HQLA on the balance sheets of commercial bank agents.

5.2.3 Impact on the Transmission of Monetary Policy

The setting of key interest rates by the central bank is an important tool of monetary policy. In our model, monetary policy is conducted exclusively through the marginal lending rate r^{CB}, which specifies at which interest rate the central bank offers overnight credit to banks. Figure 8(a) shows how a sudden 1 percentage point change in the marginal lending rate (monetary policy shock) leads to a change in the average volume of loans commercial bank agents provide to the real sector. The lines track the percentage difference in the average loan supply of the median simulation run between the unshocked and the shocked system. As expected, loosening monetary

profitability level under the LCR, on the other hand, is not a purely emergent phenomena. It is partly due to our modeling choice of not allowing commercial bank agents to endogenously change the interest rate on loans to the real sector. This makes the model prediction that profitability will suffer under liquidity regulation weaker than the prediction that interest rates of long term wholesale debt will increase. Banks could restore their profitability in the light of higher funding costs by increasing the interest rate they charge on loans. We refrain from allowing such adjustments in the current setup because the real sector is not endogenous and can itself not react to changing interest rates. Nevertheless, we assume that the decline in volume of loans provided to the real sector under the LCR regulation (see Table 1) holds, since rising interest rates on loans would reduce the demand for loans.

policy leads to an increase in the loans volume, while a tightening leads to a decline in loans to the real sector. Note that it takes about two years before the average loan supply of commercial bank agents stabilizes at a new level, which is approximately 2.5% higher or 1% lower than the loan supply of the unshocked system. Although the monetary policy shocks take full effect only after a substantial time span, changing the marginal lending rate proves to be an effective tool under the benchmark setup. In contrast, a change in the marginal lending rate is largely ineffective under the LCR setup, as illustrated in Figure 8(b). For the first half year after the shock, there is on average no impact of the policy shock on the loan supply. Thereafter the loan supply does deviate from the unshocked system. While the direction of the deviation is largely consistent with the results from the benchmark setup, the size of the impact is economically insignificant (note the scaling factor of 10^{-3}) under the LCR setup.



(a) impact on the benchmark setup

(b) impact on the LCR setup

Figure 8: Change in the loan supply of commercial bank agents in response to a change in the marginal lending rate.

In order to understand the transmission of changes to the marginal lending rate under the benchmark setup and the apparent breakdown of the transmission channel under the LCR setup, we simulate the two setups for different values of r^{CB}. In Figure 9, we report the average values (over time and agents) of crucial variables. Figure 9(a) confirms the results from the monetary policy shock experiment above: As the marginal lending rate decreases, the average loan supply of the average commercial bank agent increases. While the relationship is pronounced and convex under the benchmark setup, it is rather weak and linear under the LCR setup. Figure 9(b) shows why the transmission differs between the two setups. It illustrates how different values for r^{CB} change the composition of wholesale debt under the benchmark setup (the blue and yellow lines). Low marginal lending rates go along with a higher fraction of overnight interbank debt and a lower fraction of longer term bonds. On the other hand, no such restructuring of wholesale debt can be observed under the LCR setup.

Because the marginal lending facility is an alternative to overnight interbank credit, the interbank market is part of the transmission channel of monetary policy. Lower values for r^{CB} therefore have an impact on the overnight interbank interest rates. The yellow line in Figure 9(d) shows how short term interest rates decrease with decreasing r^{CB}. The positive relationship between the two competing short term refinancing instruments emerges through the interaction of agents on the interbank market. Commercial bank agents are inclined to increase their share of central bank funding as the marginal lending rate decreases. In order to compete with the central bank's facility, investment bank agents need to cut short term interbank interest rates. The transmission does not end here. Interest rates of longer term wholesale debt (the blue line of Figure 9(d)) also adjust, because they compete with short term wholesale debt. In essence, monetary policy is transmitted through a chain of competing funding instruments. Under the benchmark setup, a lower r^{CB} will decrease funding costs, which will free resources to expand the loan supply of commercial banks. Under the LCR setup, on the other hand, the short term interbank market breaks down. The first link in the chain of competing funding instruments is broken, which interrupts the transmission channel of a monetary policy conducted through changes in the marginal lending rate.[34] What is left of the short term interbank market in the LCR setup, does behave similarly to the benchmark setup as can be seen in Figure 9(c), but the absolute volume is too small to keep the described transmission channel alive. Nevertheless, we observe a weak reaction of loan supply (in Figure 9(a)) and bond interest rates (in Figure 9(d)) to changes in the marginal lending rate. The transmission in this case goes through the expected default probability of commercial bank agents. Figure 9(e) plots the mean expectation by investment bank agents of the default probability of commercial bank agents.[35] Under both setups, the expected default probability tends to increase with an increasing marginal lending rate. The transmission channel works as follows: Consider a bank under stress that is partially cut-off from wholesale funding markets. This bank will have to balance its balance sheet by tapping into the marginal lending facility of the central bank, which functions as a lender of last resort. The higher the marginal lending rate, the higher the probability that the bank will remain stressed, since refinancing costs eat away at equity. Since deleveraging takes time, lower levels of equity will result in higher default probabilities, which has an impact on bond interest rates and loan supply. Figure 9(f) shows another point on the default probability distribution. When looking at median values, a difference between the two setups becomes evident. While the median expected default probability under the benchmark setup decreases with increasing r^{CB}, the opposite is true for the LCR setup. The median, in contrast to the mean, excludes stressed agents with high default probabilities. The figure therefore implies that the typical commercial bank agent under the benchmark setup will choose to increase the risk on its asset side when monetary policy is decreasing interest rates. In our model, commercial bank agents increase their risk by expanding their balance sheet (i.e. increase leverage). In reality, banks could, of course, also shift into riskier assets. From the commercial bank agent's perspective, a riskier asset side, whether through higher leverage or high-risk investments, seems to be a sensible decision. When interest rates are persistently low, the risk from changes to wholesale refinancing costs will decrease and create risk-bearing capacity on the asset side. With other words, our model implies a risk channel of monetary policy. Commercial bank agents become more vulnerable to unexpected shocks to wholesale funding costs. This channel is not apparent under the LCR

[34] In our framework, the marginal lending facility is part of the interbank market (see Eq. 3.16). Because the treatment of central bank loans and overnight interbank loans differs under the LCR regulation, this modeling choice can be problematic. Specifically, while the run-off rate for overnight interbank loans is 100%, it is 0% for central bank credit (national regulators may demand higher rates). Taking this into account, banks could very well fund their required HQLA with short term central bank loans instead of long term wholesale debt. However, it is unclear if such behavior could completely restore the transmission of monetary policy. If banks start relying more on central bank credit to finance HQLAs (Rezende et al. (2016) find that this is the case for banks that are subject to the LCR in the U.S.), it may become more difficult for the central bank to manage the amount of liquidity in the banking system with conventional tools. On the other hand, if the central bank does not reliably satisfy the increasing demand for central bank liquidity (the LCR was in part introduced to reduce the dependence on the central bank), banks will, as our results suggest, seek to replace short term funding (including borrowing from the central bank) with long term funding, breaking the link between central bank and wholesale interest rates.

[35] The expected default probabilities only consider the risk on the asset side (see Eq. (3.58)). Refinancing risks of commercial bank agents are neglected by investment bank agents.

setup.

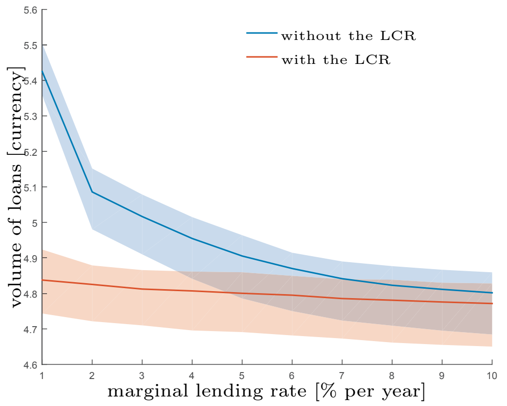
On a general note, we are well aware of the fact that monetary policy in reality is multidimensional and includes more approaches than the setting of short term interest rates. Longer term refinancing operations by the central bank, for example, will still be able to influence market rates. Furthermore, the transmission channels of open market operations and unconventional monetary policy are unlikely to be strongly affected by the LCR regulation. Nevertheless, the simulations above are not invalid. They suggest that the transmission of a certain type of monetary policy is likely to be suppressed when the LCR regulation is binding. Furthermore, much of the impact of monetary policy depends on the interaction between the financial sector and the real sector. These interactions are not included in our current framework. For example, loose monetary policy is expected to stimulate consumption, while monetary tightening typically drains customer deposits from the banking sector. It has recently been suggested by New York Fed researchers that the LCR regulation could help to strengthen the so called bank lending channel (see Choi and Velasquez, 2016). They argue that since wholesale debt has become a common funding source for commercial banks, a monetary policy induced decrease in deposits would quickly be replaced by wholesale funding, while an increase in deposits would lead to a reduction in wholesale funding. The result of this behavior is a reduced effect of monetary policy on bank lending. We can test their hypothesis of a more pronounced bank lending channel under the LCR regulation by simulating a shock on deposits and by comparing the impact on loan supply for our two setups. For the benchmark setup, Figure 10(a) shows the change in the loan supply compared to the unshocked system for a 10% increase and decrease in customer deposits. Clearly, the loan supply increases when customer deposits increase and vice versa. However, without any compensation of the shock through wholesale debt, we would expect lending to increase or decrease by approximately 3%, which is far higher than the observed change in average loan supply.[36] The fact that a bank lending channel is observed at all, is due to the different characteristics of deposits and wholesale funding. Since deposits, at least within our framework, constitute a cheaper and more stable source of funding than wholesale debt, increasing them allows for a small balance sheet expansion, while a decrease in customer deposits leads to a slight contraction of the balance sheet. In order to show the impact of the LCR on loan supply, Figure 10(b) plots the cumulative difference between the shock responses under both setups. At each point in time, the cumulative difference states how much more or less loans (in percent) would have been supplied in total under the LCR setup since the unexpected change in deposits.[37] Figure 10(b) confirms the hypothesis of Choi and Velasquez (2016). The bank lending channel is more pronounced under the LCR regulation. However, large impulses for the real economy are not to be expected. At its best, i.e. in the first months after the shock, commercial bank agents provide approximately 0.05% more or less loans under the LCR setup than under the benchmark setup. In the longer run, the effect on the bank lending channel remains positive, but loses intensity.

[36]As reported in Table 1, customer deposits make up approximately 30% of total assets or loans to the real sector. Without any compensation, an increase or decrease of customer deposits by 10% should therefore increase or decrease total assets by 3%, respectively.

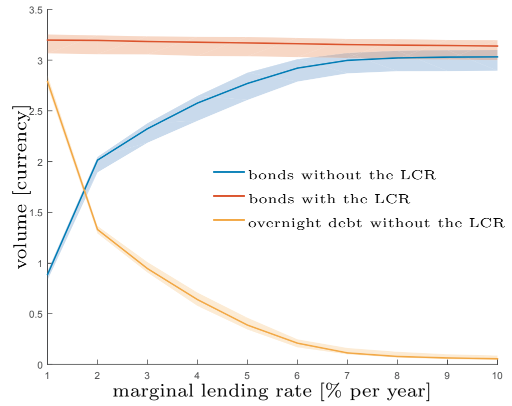
[37]The cumulative difference CD_t is computed as follows:

$$\mathrm{CD}_t = \frac{\left(\sum_{\tau=1}^{t} \Delta L_\tau^{\mathrm{LCR,s}} L_\tau^{\mathrm{Bench,u}}\right) - \left(\sum_{\tau=1}^{t} \Delta L_\tau^{\mathrm{Bench,s}} L_\tau^{\mathrm{Bench,u}}\right)}{\sum_{\tau=1}^{t} L_\tau^{\mathrm{Bench,u}}},$$

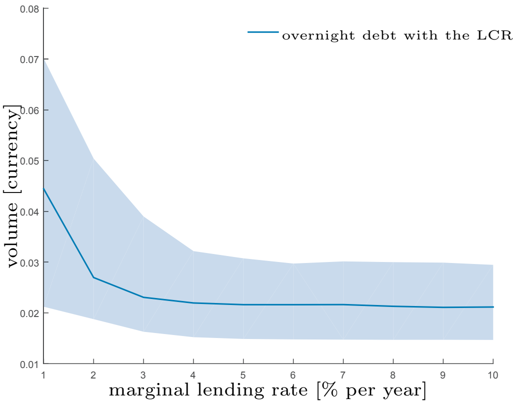
with $\Delta L_\tau^{\mathrm{LCR,s}}$ and $\Delta L_\tau^{\mathrm{Bench,s}}$ being the percentage difference of the average loan supply for the shocked LCR setup and benchmark setup, respectively. $L_\tau^{\mathrm{Bench,u}}$ denotes the average amount of loans supplied for the unshocked benchmark setup. With such a calculation of the cumulative difference, we assume that the loan supply in the unshocked case is equal for the benchmark setup and the LCR setup.



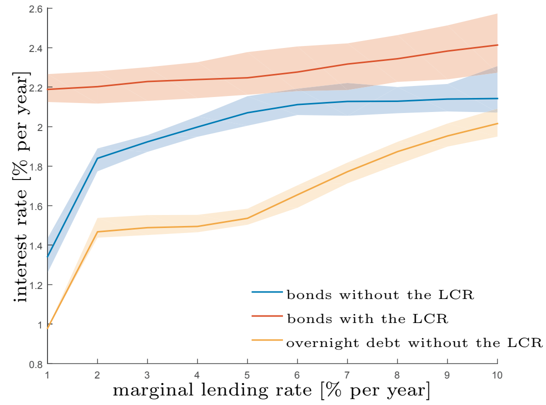
(a) impact on loans to the real sector



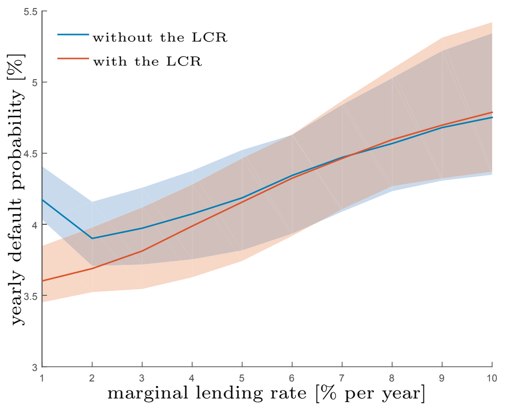
(b) impact on the volume of wholesale debt



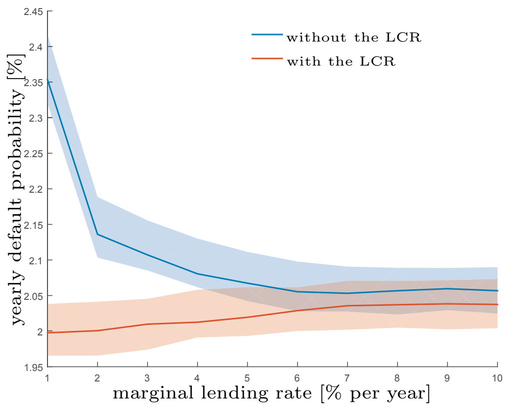
(c) impact on overnight debt in the LCR setup



(d) impact on the cost of wholesale debt



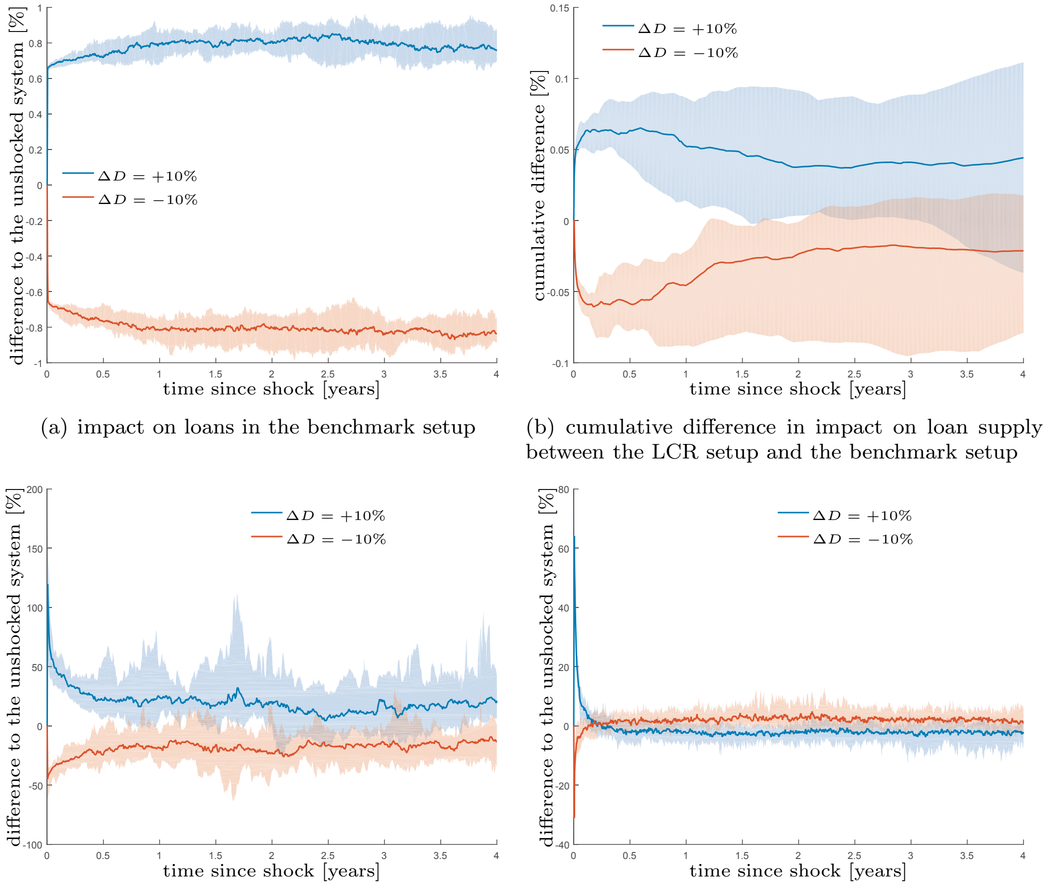
(e) impact on the expected default probability (mean)



(f) impact on the expected default probability (median)

Figure 9: The impact of different marginal lending rates on balance sheet variables, interest rates and expected default probabilities

Figures 10(c) and 10(d) illustrate the main mechanism through which the bank lending channel is strengthened under the LCR setup. It depicts the change in cash holdings in response to the shock to customer deposits. The left panel shows the response in the benchmark setup. When deposits increase, cash holdings shoot up, while they sharply decrease when deposits are withdrawn. Since it takes some time for agents to cut back on wholesale funding in case of increased deposits, the additional cash is gradually reduced. On the other hand, a sudden reduction in deposits will first deplete cash holdings until sufficient wholesale funds can be raised for compensation. The impact of changes in cash on other balance sheet variables is, however, very limited, since the cash to total assets ratio in the benchmark setup is on average lower than 0.5% (see Table 1). Figure 10(d), which graphs the impact on cash under the LCR setup, is more interesting. Here, a positive shock on deposits eventually leads to a reduction in cash holdings, while a negative shock eventually increases cash. The reason for this dynamic lies in the specification of the LCR. Customer deposits are assigned a run-off rate of 3% because they are deemed a rather safe funding instrument by the regulator. Unsecured wholesale debt, on the contrary, is deemed a very risky funding instrument. The regulator assumes that in times of stress banks will not be able to refinance maturing unsecured wholesale debt, which translates into a run off rate of 100%. Consequentially, the larger the proportion of assets funded by run-prone wholesale debt, the more HQLA banks need to hold. As explained in Section 5.2.1, higher volumes of HQLA will partially crowd out loans in the LCR setup. Lower required holdings of HQLA, on the other hand, will add to a bank's capacity to provide loans to the real sector. Through this mechanism, the LCR regulation may indeed contribute to a strengthening of the bank lending channel. However, taking into account our finding that the transmission of changes to the marginal lending rate is markedly disturbed by the LCR regulation, it remains uncertain whether monetary policy will become more or less effective in the future. In order to assess the cumulative effect, it would be necessary to endogenize the real sector in our model. What our analysis can nevertheless show, is how the nature of monetary policy transmission is expected to change due to the introduction of liquidity regulation.



(a) impact on loans in the benchmark setup

(b) cumulative difference in impact on loan supply between the LCR setup and the benchmark setup

(c) impact on cash in the benchmark setup

(d) response to cash in the LCR setup

Figure 10: The impact of a shock to the volume of customer deposits on loans to the real sector and cash on commercial bank agents balance sheets.

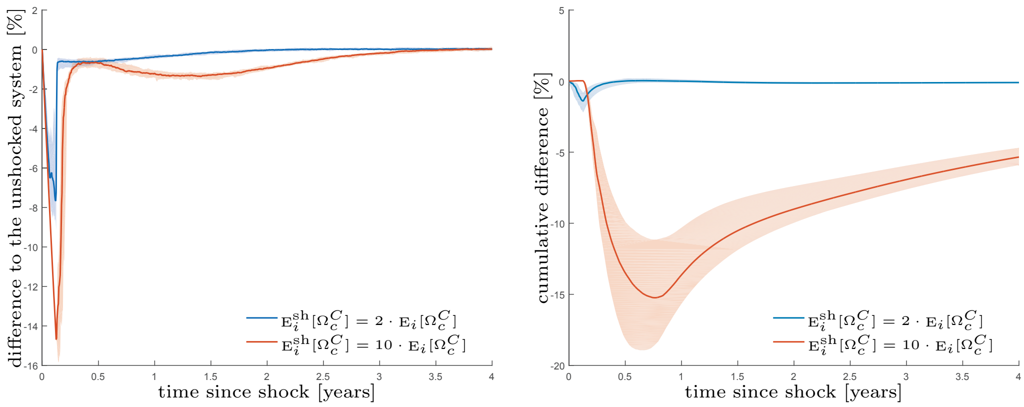
5.2.4 Impact of a Confidence Shock

Our analysis so far has shown that a binding LCR regulation will pressure banks to make changes to the structure of their balance sheet. The principal aim of the regulator is that these changes will contribute to stabilizing the financial system. In order to filter out the impact of the regulation on the stability of the system, we compare the dynamics that are triggered by a controlled shock in the benchmark setup and the LCR setup.[38] The focus of the following analysis lies on explaining

[38]Financial stability in the context of the LCR regulation does not feature prominently in the literature. One exception is van den End and Kruidhof (2013), who argue for a flexible LCR requirement in order to mitigate negative side effects such as fire sales during times of stress. Relatedly, many observers have raised concerns that even though the regulator explicitly allows banks to temporarily fall below minimum requirements when stressed, they may be reluctant to do so in reality (see e.g. Stein, 2013). Similar to the stigma associated with accessing the discount window (see e.g. Armantier et al., 2015), banks may fear a loss of reputation when having to report that their LCR falls short of 100%. In our model, commercial bank agents draw down their HQLA in times of stress. Technically, whenever they fail to refinance wholesale debt and need liquidity assistance from the central bank, they reduce their HQLA before accessing the marginal lending facility. There are no reputational costs associated with this behavior.

the impact of the LCR regulation on the stability of banks' loan supply to the real sector.

The default of Lehman Brothers in September 2008 led to a surge in uncertainty about the solvency of banks. We want to model a mutual loss of confidence in the banking sector by shocking the expectations investment bank agents have about the default probabilities of commercial bank agents. Specifically, we multiply the expectation $\mathrm{E}_i[\Omega_c^C]$ (see Eq. (3.59)) with a factor to obtain the shocked expectation $\mathrm{E}_i^{\mathrm{sh}}[\Omega_c^C]$, which lasts for 30 trading days before returning to its normal level. The left panel of Figure 11 shows the impact of the shocks on the average loan portfolio of commercial bank agents in the benchmark setup. Two intensities of the confidence shock are plotted. The blue line graphs the percent difference between the shocked and unshocked system when default probability expectations are doubled, while the red line draws that difference for a tenfold increase in expected default probabilities. For both shock intensities, the loan supply decreases immediately after the shock and starts to rise again after 30 days. However, the decrease of loan supply as well as its subsequent recovery is steeper for the weaker shock than for the stronger shock. Noteworthy is also the belly that the loan supply displays for the severer shock, which lasts for approximately three years (from 0.5 to 3.5 years after the shock). It implies that short lived but intense confidence crises in the financial sector can have a sustained effect on the real economy. The implementation of the liquidity coverage ratio aggravates the adverse effects of confidence shocks on the loan supply, as illustrated in Figure 11(b). While the cumulative difference (computed as specified in Footnote 37) between the loan supply in the benchmark setup and the LCR setup quickly becomes irrelevant for the weak confidence shock, it is substantial for the larger shock. One year after the shock, commercial bank agents in the LCR setup provided almost 15% less loans to the real sector than their counterparts in the benchmark setup. The consequence is likely to be a severe recession.



(a) impact on loans in the benchmark setup

(b) cumulative difference in impact on loan supply between the LCR setup and the benchmark setup

Figure 11: The impact of a confidence shock on the supply of loans to the real sector.

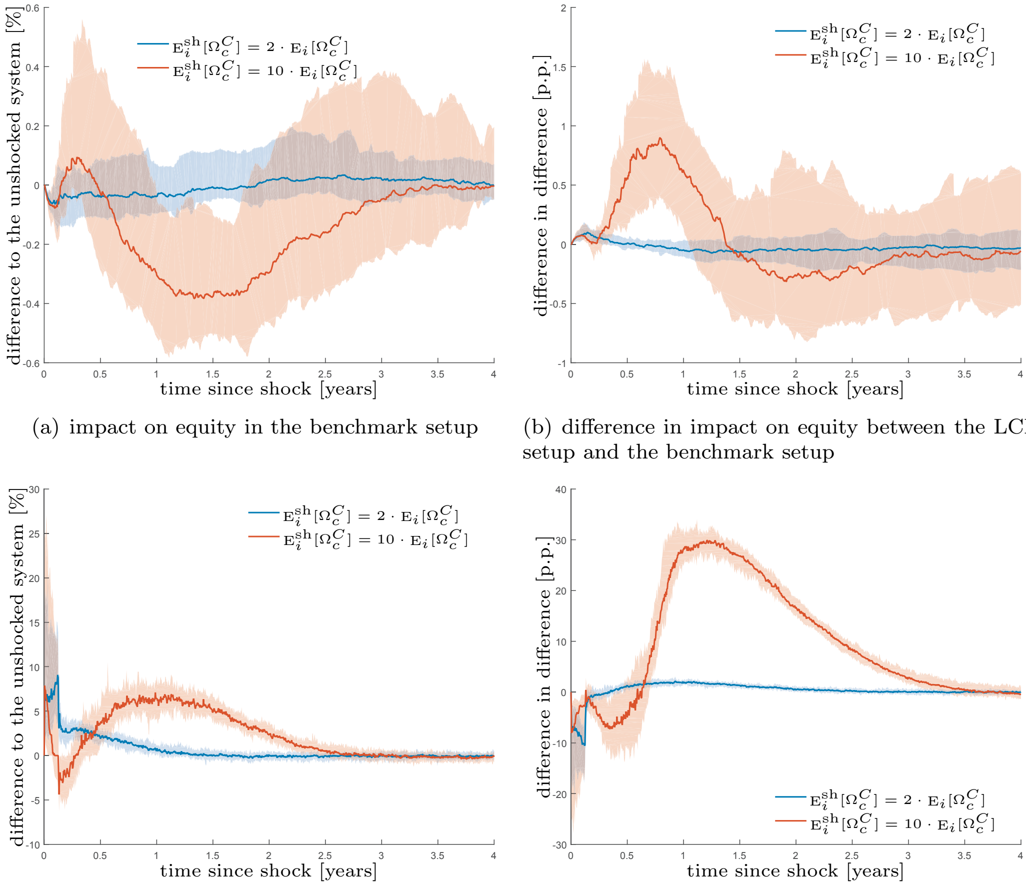
Although the temporary loss of confidence in the solvency of commercial bank agents triggers the decline in loan supply, the stability of commercial bank agents is not compromised as a consequence of the shock.[39] Figure 12(a) shows that the extent to which commercial banks' equity in the shocked benchmark scenario deviates from the unshocked system is rather small. The deviation is explained by changing funding costs, depicted in Figure 12(c). Investment

[39]It is important to note that the lack of any feedback between the real sector and the financial sector is an issue here. Typically, recessions are accompanied by a deterioration of credit quality, which would have an impact on the solvency of commercial banks.

bank agents react to their perception of higher default probabilities by increasing interest rates of wholesale debt. Since short term debt needs to be rolled over quickly, higher interest rates immediately show up in funding costs in the benchmark setup and cause the initial drop in equity seen in Figure 12(a). Then, as commercial bank agents shrink their balance sheet by reducing their loan supply, average funding costs temporarily decline, which increases profitability and hence equity. After the shock is resolved and default probability expectations normalize, investment bank agents reduce the interest rates on wholesale debt. However, for some months they remain higher than they would have been without the shock (see bond prices in Figure 15(a)). In part, this is the case because unexpected events influence the risk assessment of agents by raising their awareness about the potential faultiness of their expectations.[40] The increased cost for wholesale debt explains why approximately half a year after the shock the equity level of commercial bank agents falls below the level measured in the unshocked system. Note that because payment conditions for wholesale debt are defined for its entire duration, average funding costs return to their normal (unshocked) level after three years, while bond prices already normalize after approximately ten months.

Unlike its impact on loan supply, the LCR regulation has a positive effect on commercial bank agents' equity for the first one and a half years after the shock. Figure 12(b) graphs the percentage point difference of the changes in equity capital with respect to the unshocked system between the LCR setup and the benchmark setup. The differences in the two impacts of the shock on equity can be explained by looking at the difference in changes to the funding costs under the two setups, which is illustrated in Figure 12(d). A higher share of long term wholesale debt under the LCR setup implies that less debt needs to be rolled over when interest rates sharply increase in response to the confidence shock. Therefore, average funding costs rise less quickly when commercial banks comply with the LCR regulation. This is despite the fact that bond prices fall below their counterparts in the benchmark setup (see Figure 15(b)). However, as soon as commercial bank agents stop deleveraging and start expanding their loan portfolios, the higher bond interest rates increase overall funding costs. Consequentially, equity falls under the level displayed in the benchmark setup.

[40]Technically, the increased prudence in response to a shock is introduced through the inclusion of past forecast errors into investment bank agents' expectations of variance (see Eq. (3.63). The parameter ψ^B thereby insures that large misjudgments remain in memory for some time.



(a) impact on equity in the benchmark setup

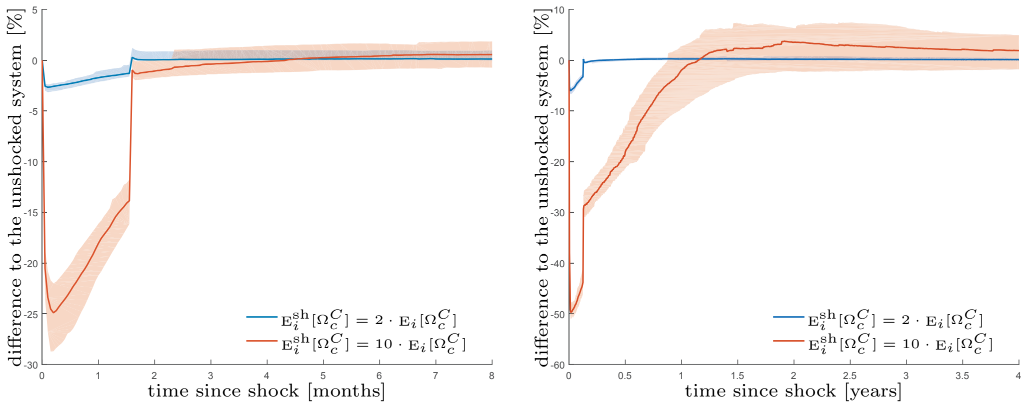
(b) difference in impact on equity between the LCR setup and the benchmark setup

(c) impact on refinancing costs in the benchmark setup

(d) difference in impact on funding costs between the LCR setup and the benchmark setup

Figure 12: The impact of a confidence shock on commercial bank agents' equity and average funding costs.

Commercial bank agents' equity levels and funding costs apparently do not explain the detrimental effect the liquidity coverage ratio regulation has on the loan supply to the real sector. The dynamics of equity of investment bank agents are more instructive. If investment bank agents' balance sheets remain unconstrained during the shock, we would expect that the symmetry of the confidence shock (i.e. the initial increase in expected default probability is fully reversed after 30 periods) will lead to short lived implications of the shock. Any initial detrimental effect should be followed by a beneficial effect of similar magnitude as expectations of commercial bank agents' default probabilities normalize. Indeed, this is what we find under the benchmark setup. Figure 13(a) plots the impact of the confidence shocks on investment bank equity under the benchmark setup. When expectations of commercial bank agents' default probability increases, investment bank agents suffer valuation losses. In case of the large shock, equity decreases by almost 25% at first, but starts to recover immediately with increasing bond prices (see Figure 15(a)). As confidence is restored after 30 days, equity reaches a level that is only slightly below the level measured in the benchmark setup without the shock. After approximately four months any trace of the confidence shock disappears. Although the recovery of investment bank agents' equity under the large confidence shock is rather swift, there is a noticeable relation between the speed of the recovery and the shock size. This relation is more salient under the LCR setup. Figure 13(b) shows that while the impact of the small shock on equity appears very similar under both setups (it is slightly worse under the LCR setup), the impacts of the large shock have a different quality in the two setups. Under the LCR setup, the initial drop in equity is about 25 percentage points deeper and the resolution of the shock after 30 days lifts equity to a level that is still almost 30 percent below its unshocked counterpart. It takes more than a year before equity reaches and then surpasses the level measured under the LCR setup without the confidence shock. The stronger initial decline in equity can be explained by a stronger decline in bond prices, which drop on average approximately 8 percentage points below their counterparts in the benchmark setup (see Figure 15(b)). However, bond prices alone do not explain the qualitative difference in the impact on equity between the two setups. In particular, it needs to be explained why the symmetry between the initial detrimental effect of the shock and the subsequent beneficial effect of its resolution is broken.

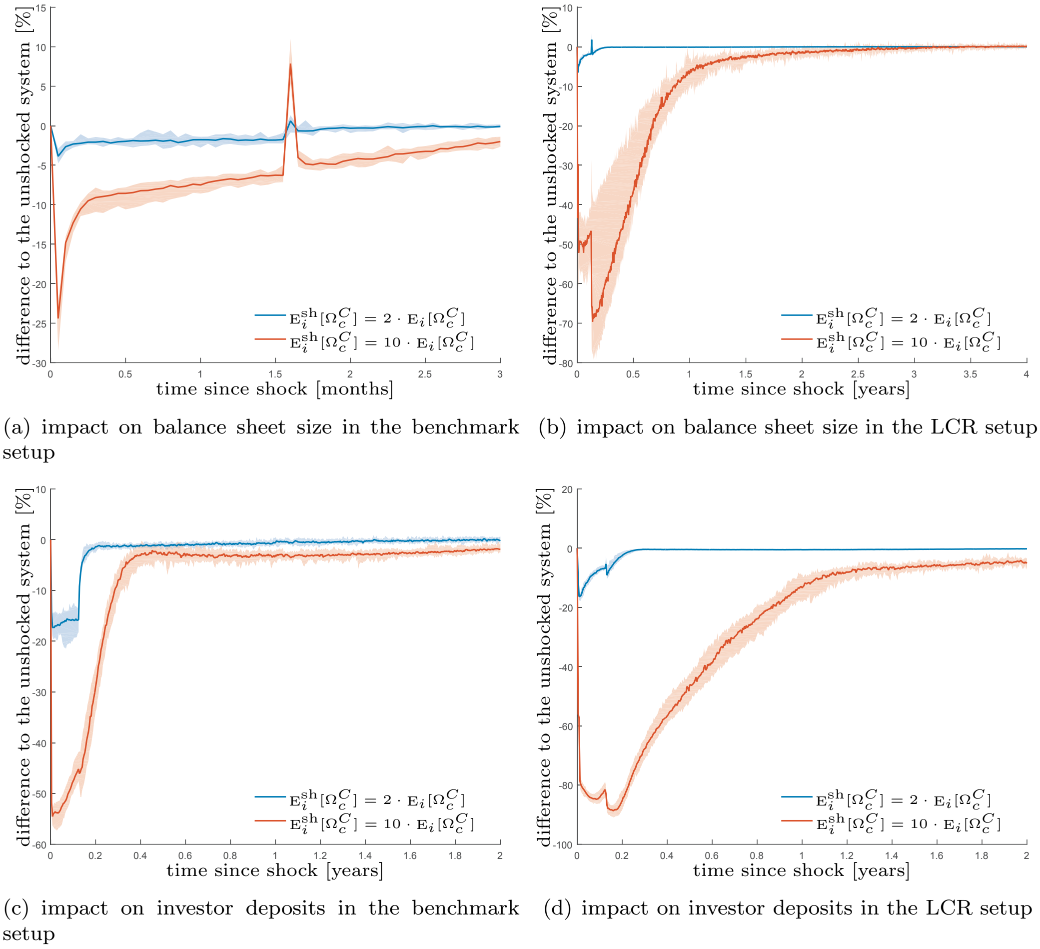


(a) impact on equity in the benchmark setup

(b) impact on equity in the LCR setup

Figure 13: The impact of a confidence shock on the equity of investment bank agents.

Figure 14(a) plots the average balance sheet size of investment bank agents in response to the confidence shock in the benchmark setup. Due to the initial negative valuation effect, investment bank agents start deleveraging, which leads to the observed contraction of balance sheets. The negative and positive peaks mark the first and last period of the shock, respectively. They are caused by overreactions of agents due to a temporary mispricing of assets. Nevertheless, the size of investment bank agents' balance sheets quickly recovers. Figure 14(b) shows, on the other hand, that when banks comply with the LCR regulation the average size of balance sheets is first halved when the shock hits and then further decimated as the shock is resolved after 30 periods. While the first contraction of balance sheet size is a deliberate reaction to the decline in equity, the second seems counterintuitive. When expectations about commercial bank agents' default probabilities normalize, the positive valuation effect induced by rising bond prices raises equity and should thereby contribute to a normalization of balance sheet size. However, the fact that investment bank agents have sold most of their portfolio in the wake of the shock dilutes the positive valuation effect from rising bond prices. At the same time, the price changes induced by both the shock and its resolution increase the volatility of investment bank agents' earnings, which raises concerns among investors and leads them to withdraw their deposits, as illustrated in Figure 14(d). The low level of funding at the time when the confidence shock is resolved furthermore has a peculiar effect on the prices of nb-securities. While they increase sharply after 30 days under the benchmark setup, they drop in the LCR setup (see Figure 15(c) and 15(d)). The reason for this can be derived from the portfolio optimization of investment bank agents. Specifically, when the confidence shock is resolved, bank bonds become undervalued. As a result, investment bank agents shift their scarce funding out of nb-securities and into bank bonds, which leads to the drop in nb-security prices. The valuation effect of this price drop further dilutes the positive valuation effect of rising bond prices. Our analysis suggests that the larger the confidence shock, the higher the asymmetry between the initial adverse effect of the shock on investment bank agents and the ensuing beneficial effect when the shock is resolved. The circumstances that explain this growing asymmetry are accelerating investor deposit withdrawals, i.e. constrained balance sheets, and an increasing dilution of positive valuation effects.



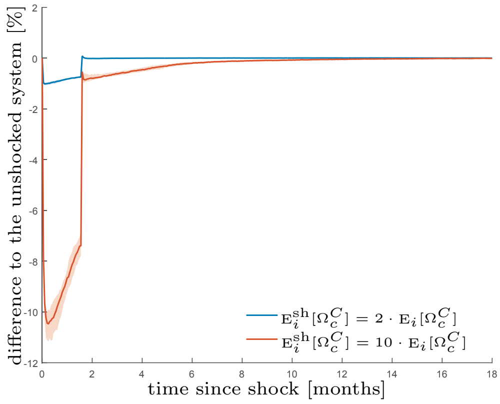
(a) impact on balance sheet size in the benchmark setup

(b) impact on balance sheet size in the LCR setup

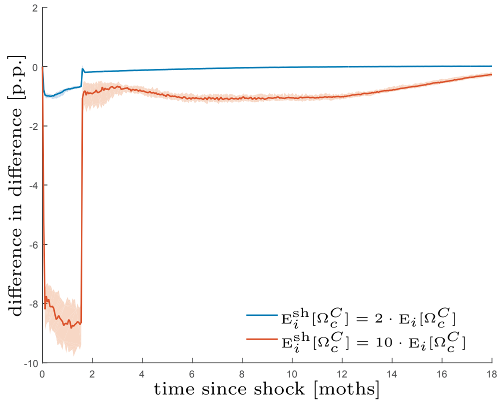
(c) impact on investor deposits in the benchmark setup

(d) impact on investor deposits in the LCR setup

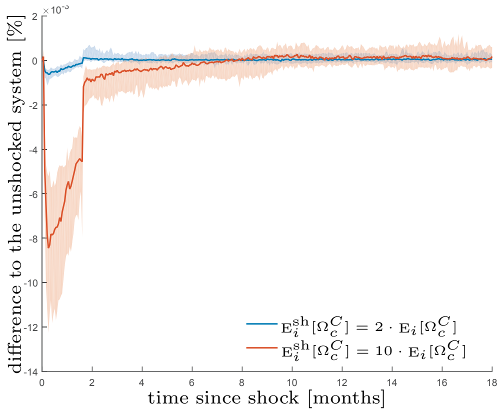
Figure 14: The impact of a confidence shock on investment bank agents' balance sheet size and on the volume of investor deposits.



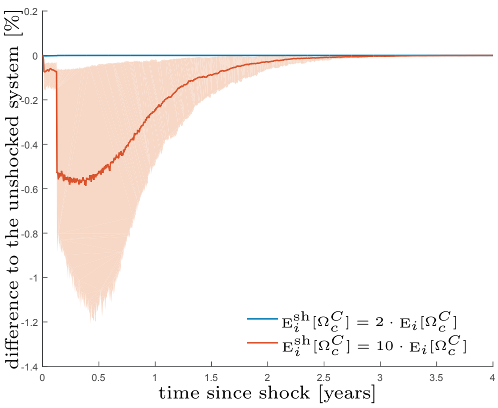
(a) impact on bond prices in the benchmark setup



(b) difference in impact on bond prices between the LCR setup and the benchmark setup



(c) impact on nb-security prices in the benchmark setup

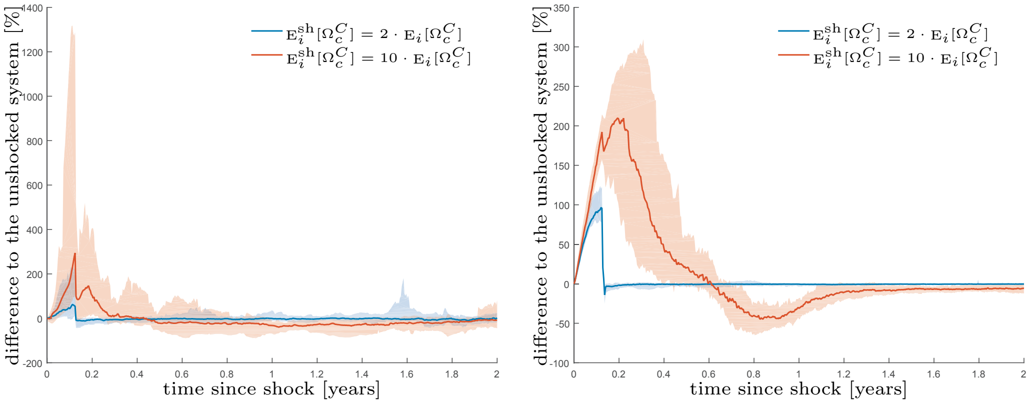


(d) impact on nb-security prices in the LCR setup

Figure 15: The impact of a confidence shock on average bank bond prices and average nb-security prices.

Under the LCR setup and large enough confidence shocks, the asymmetry becomes increasingly destabilizing, at least with regard to commercial bank agents' loan supply to the real sector. The mechanism behind this destabilizing effect of the liquidity regulation is simple: The LCR incentivizes commercial bank agents to increase the maturity of their wholesale funding. The increased maturity makes investment bank agents' bond holdings more prone to valuation effects, i.e. it increases risk. When shocks to commercial bank agents' perceived solvency are within the range of what investment bank agents expect, the demanded higher interest rate compensates for the additional risk. The described asymmetry, however, causes unexpectedly strong adverse shocks to have a lasting impact on investment bank agents' equity and balance sheet size. The consequential decline in the supply of cheap wholesale funding after the shock causes the subdued loan supply to the real sector. With other words, by increasing the funding stability of commercial bank agents through the LCR regulation, the regulator deepens the contagion channel. The longer the maturity of tradable wholesale debt, the more immediate will be the transfer of stress between different bank business models.

Before ending our analysis, we want to point out an interesting regularity that arises when



(a) impact on cash in the benchmark setup

(b) impact on cash in the LCR setup

Figure 16: The impact of a confidence shock on commercial bank agents' cash holdings.

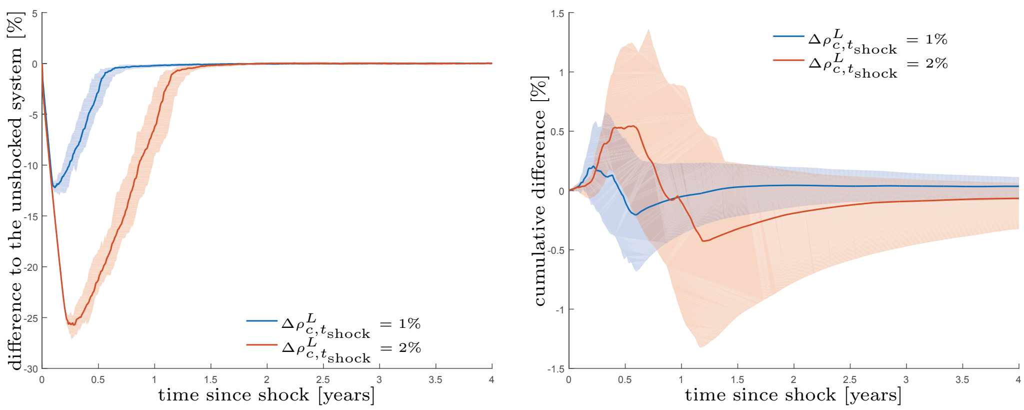
implementing the confidence shocks. Under both the benchmark setup and the LCR setup, cash holdings increase as a consequence of the shock. This seems counterintuitive, as the regulator explicitly allows banks to deviate from a LCR of 100% in times of stress in order to compensate withdrawals of wholesale funding by selling high quality liquid assets. Figure 16 shows how cash starts rising immediately after a shock hits.[41] While the resolution of the shock after 30 periods leads to a quick normalization of cash holdings in case of the small confidence shocks, the large confidence shocks have a more sustained effect on cash holdings. The effect is interesting because it mimics increasing liquidity buffers at the onset of the financial crisis in 2007 (see e.g. Acharya and Merrouche, 2012; De Haan and van den End, 2013). While the literature provides several different explanations for the observed liquidity hoarding that range from precautionary to predatory motives (see Gale and Yorulmazer, 2013; Acharya et al., 2012, , respectively), in our model, the growth of cash holdings is an indirect and rather trivial consequence of deleveraging activities. Specifically, when wholesale funding costs sore in response to the confidence shock, commercial bank agents start shrinking their balance sheets. Maturing loans are not renewed, while inflows are used to pay back wholesale debt. Because loan portfolios can be reduced faster than wholesale debt, cash accumulates until it can be used to pay back wholesale debt.[42] Note that the system wide increase in liquidity is only plausible under two related conditions: first, the confidence shock affects the banking system as a whole and second, short term money markets are disturbed. These conditions are fulfilled, both in reality during the financial crisis 2007-2009 and when we shock our artificial financial system. If the shock would be idiosyncratic and money markets fully functional, excess liquidity would not accumulate on balance sheets, but would be issued as short term wholesale debt.

[41]The very large percentage differences in cash holdings relative to the unshocked system, especially under the benchmark setup, are due to very low initial average cash holdings.

[42]It seems reasonable to assume that in reality loan portfolios can shrink faster than wholesale debt. This can be the case even when the average maturity of wholesale debt is shorter than the average maturity of loans. The necessary condition for this to happen in our model is given by the following relation: $(1 - m_L)L_{c,t} > (1 - m_B)B_{c,t} + I_{c,t}$. The fact that commercial banks finance a considerable part of their loan portfolio with deposits and overnight interbank debt is only a small fraction of wholesale debt makes the fulfillment of the condition plausible.

5.2.5 Impact of a Solvency Shock

Our analysis suggests that the banking system under the LCR regulation will amplify the adverse effects of a confidence shock, especially a large one. However, there are plenty of other shocks that could hit the banking sector, for which the LCR regulation could prove to be beneficial. In the following, we take a look at the impact of a solvency shock on the financial system. Specifically, we manipulate the exogenous loan default rate $\rho^L_{c,t_{\text{shock}}}$ at time $t = t_{\text{shock}}$ by adding a factor $\Delta\rho^L_{c,t_{\text{shock}}}$. The shock lasts for exactly one period after which loan default rates return to their normal levels. Again we show the impact of two different shock sizes. In the following figures, the blue lines graph the impact of a one time increase in the loan default rate by one percentage point, while the red lines show the dynamics of a two percentage point increase.



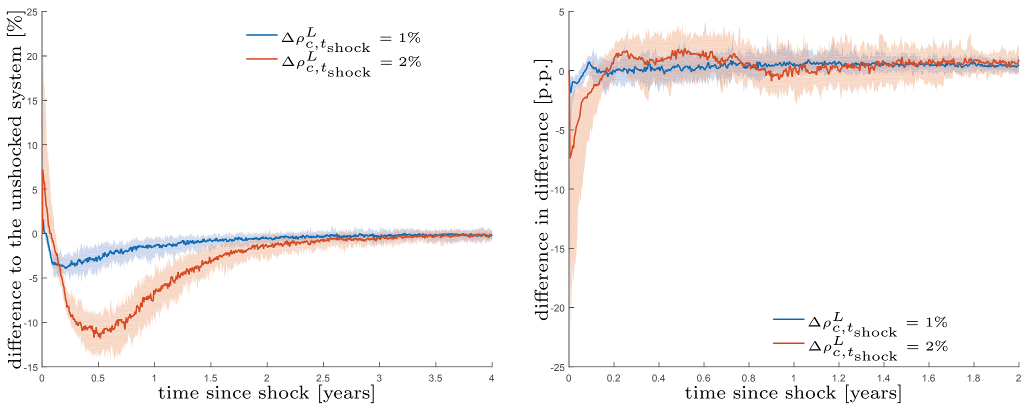
(a) impact on loans in the benchmark setup

(b) cumulative difference in impact on loan supply between the LCR setup and the benchmark setup

Figure 17: The impact of a solvency shock on the supply of loans to the real sector.

Figure 17 shows the impact (relative to the unshocked system) of the shocks on the loan supply of commercial bank agents under the benchmark setup. Note that the reductions in loan supply are more than ten times larger than the initial shocks. This is the case because the drop in equity induced by the shock leads to a surge in leverage, which is unacceptable to the risk management of agents. The recovery sets in only after the consequential deleveraging process has been completed. Unsurprisingly, the length of the recovery is positively related to the size of the shock. While it takes the system about 9 months to recover from the 1% shock, the length of the recovery doubles for the 2% shock. The comparison of the solvency shocks' impact on loan supply under the benchmark setup and the LCR setup, depicted in Figure 17(b), has two distinctive features. During the first couple of months, the cumulative (computed as detailed in Footnote 37) loan supply is larger when the LCR is implemented. After some time, the cumulative loan supply in the LCR setup falls below that of the benchmark setup and then approaches the zero-line as time progresses. It is important to notice that for both shocks, the difference between the two setups is rather small. For example, assuming that the loan portfolios prior to the shock are equally large under both setups, the cumulative loan supply half a year after the larger shock is merely 0.5% higher in the LCR setup than in the benchmark setup. After approximately 14 months, on the other hand, commercial bank agents under the LCR setup have provided about 0.5% fewer loans to the real sector than under the benchmark setup. Four years after the larger shock, the cumulative difference in loan supply between both setups has been reduced to an economically rather insignificant negative 0.05%.

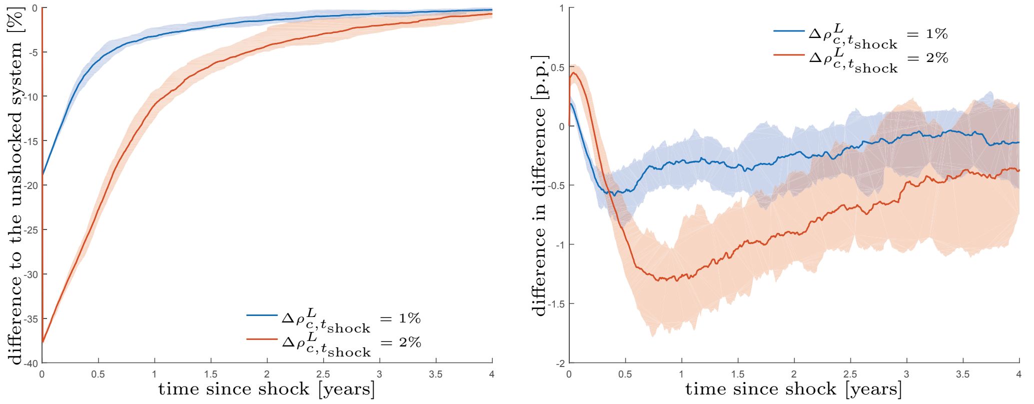
The explanation for the difference in impact of the solvency shocks between the two setups can be traced back to two phenomena. The initial positive impact can be ascribed to the higher funding stability that the LCR regulation evokes. Figure 18 shows that while average funding costs change substantially in response to the shock, the difference in the responses between the two setups is large only in the immediate aftermath of the shocks. Because short term funding is curtailed by the LCR regulation, commercial bank agents do not roll over overnight interbank debt at higher interest rates, which is beneficial to equity and slightly reduces deleveraging pressure. The dynamics of equity reveal the phenomena that eventually leads to the overall detrimental (albeit minor) impact of the LCR regulation on loan supply. Figure 19(a) plots the impact of the solvency shocks on equity under the benchmark setup. Equity drops sharply at first and then slowly recovers as time progresses. The difference in impacts between the LCR setup and the benchmark setup, shown in Figure 19(b), is positive at first, but quickly turns negative. The initial beneficial effect can be traced back to the higher funding stability under the LCR, while the subsequent detrimental effect on equity implies a slower recovery when commercial bank agents comply with the LCR regulation. Equity recovers at a slower pace because the LCR reduces the profitability of commercial bank agents by increasing average wholesale funding (see Figure 6 and Table 3). As we already mentioned in Section 5.2.2, a reduction in banks' profitability due to the LCR regulation is not entirely an emergent phenomenon in our model. In order to maintain a constant profit rate, banks could, and probably would, increase interest rates on loans in response to rising funding costs. Under these circumstances, the loan supply could recover at equal rates in the two setups, which would lead to a stabilizing effect of the LCR regulation on loan supply. On the other hand, higher interest rates on loans are likely to reduce loan demand.



(a) impact on refinancing costs in the benchmark setup

(b) difference in impact on funding costs between the LCR setup and the benchmark setup

Figure 18: The impact of a solvency shock on the average funding costs of commercial bank agents.



(a) impact on equity in the benchmark setup

(b) difference in impact on equity between the LCR setup and the benchmark setup

Figure 19: The impact of a solvency shock on commercial bank agents' equity.

6 Conclusion

We have developed a model of the financial system that can be used as a test bed for banking regulation. The framework comprises the agents and institutions that have proved crucial in the propagation of the subprime mortgage shock in the U.S. into a global financial crisis. Specifically, we have modeled two agent types that represent commercial banks on one side and investment banks and shadow banks on the other side. The agents of the model interact on wholesale debt markets. Beside a market for short term interbank loans and long term bank bonds, other funding sources include insured customer deposits, uninsured investor deposits, secured short term debt in the form of repos as well as the possibility to borrow securities for the purpose of short selling. Credit to the real sector is the principal asset of commercial bank agents, while investment bank agents specialize in trading securities, which may differ according to risk, maturity and liquidity. We endow agents with sophisticated tools to manage the asset and liability side of their balance sheet. Based on their expectations, agents try to behave optimally. Therefore they can quickly adapt to changing circumstances, which may arise endogenously or are enforced exogenously.

We employ the framework to assess the impact of the liquidity coverage ratio regulation on balance sheets, interest rates, monetary policy transmission and some aspects of financial stability. Our findings confirm existing impact assessments in that they suggest that the regulation will lead to a lower supply of bank loans to the real sector, higher interest rates and a shift towards longer term wholesale funding. When the LCR regulation is the binding constraint on balance sheets, a sharp decline in the role of the short term interbank market as a funding source disturbs the transmission of monetary policy. In particular, changes to short term central bank interest rates will be less effective in stimulating or curtailing the supply of loans to the real sector. On the other hand, we find that the lending channel of monetary policy through changes in customer deposits will be slightly more pronounced under the LCR regulation. Furthermore, we evaluate the impact of a confidence shock and a solvency shock on the loan supply to the real sector. A large and unexpected shock to confidence, which we model with a temporary increase in perceived default probability of commercial bank agents, leads to a severe credit crunch under the LCR regulation. While the regulation has a stabilizing effect on commercial banks, it decreases the stability of investment banks, who are the creditors of commercial banks in wholesale debt markets. A sustained decline in the supply of wholesale funding in response to the confidence shock is ultimately responsible for the credit crunch. In contrast, the LCR regulation does alleviate the immediate adverse consequences of a solvency shock on the loan supply. However, the positive effect is rather modest and short lived. Lower average profit rates of commercial bank agents lead to a slower recovery and eventually to a detrimental impact of the LCR regulation on loan supply.

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A Initialization

Random variable	**Description**	**Distribution**	**Parameters**
Commercial Banks			
$\Delta D_{c,t}$	Change in customer deposits	Normal	$\mu_{c,t}^{\Delta D} = 0, \sigma_{c,t}^{\Delta D} = 0.001$
$\rho_{c,t}^{L}$	Loan default rate	Lognormal	$\mathrm{E}_{c,t}[\rho^L] = \frac{0.04}{250}, \mathrm{Var}_{c,t}(\rho^L) = (\frac{0.4}{250})^2$
$r_{c,t}^{W}$	Refinancing costs in commercial banks' value at risk calculation	Normal	Endogenous
Investment Banks			
$\epsilon_{i,j,t}^{\pi}$	Stochastic error term in forecasts of long-term profit rates	Normal	$\mu_{i,j,t}^{\epsilon^\pi} = \frac{0.05}{250}, \sigma_{i,j,t}^{\epsilon^\pi} = \frac{0.1}{250}$
$\epsilon_{i,c,t}^{\Omega}$	Stochastic valuation error of commercial banks' default probabilities	Normal	$\mu_{i,c,t}^{\epsilon^\Omega} = \frac{0.05}{250}, \sigma_{i,c,t}^{\epsilon^\Omega} = \frac{0.1}{250}$
$r_{j,t}^{S}$	Stock returns in calculation of repo haircuts and margin requirements	Normal	Endogenous

Table 4: Distribution assumptions

Category	Symbol	Description	Value
General Simulation Parameters	n^C	Number of commercial banks	100
	n^I	Number of investment banks	30
	n^S	Number of stocks	15
	T	Simulation Periods	2000
Commercial Banks			
General Parameters	E_c^*	Equity target	0.3
Asset Side Management	$\bar{r}_{c,t}^L$	Interest on loans	$\frac{0.07}{250}$
	m_L	Maturity of loans	0.995
	x^L	Confidence level in value at risk calculations	0.995
Liability Side Management	$r_{c,t}^D$	Interest paid on deposits	$\frac{0.001}{250}$
	m_B	Maturity of long-term debt	0.995
Raising short-term debt	γ_v	Elasticity of trust between the banks	0
	γ_u	Elasticity of relative attractiveness of the interest rate	1
	Ξ^{min}	Lower bound for aggregation mechanism transaction indicator Ξ	1
	Ξ^{max}	Upper bound for aggregation mechanism transaction indicator Ξ	20
Expectation Formation	ψ^I	Memory parameter in calculation of $\hat{\mathrm{E}}_{c,t}[a^{CB}]$	0.1
	ψ^B	Memory parameter in calculation of $\hat{\mathrm{Var}}_{c,t}(r^B)$	0.1
	ψ^W	Memory parameter in calculation of $\hat{\mathrm{E}}_{c,t}[r_{total}^W]$, $\hat{\mathrm{Var}}_{c,t}(r_{total}^W)$	0.01
Investment Banks			
General Parameters	E_i^*	Equity target	4
Asset and Liabilities Management	λ_i	Risk aversion	20
	m_s	Maturity nb-securities	0.995
Short-term Interbank Loans and Bonds	γ_{vi}	Valuation elasticity of trust component	0
	γ_r	Valuation elasticity of return component	1
	γ_σ	Valuation elasticity of standard deviation component	5
	g^A	Discrimination factor in calculation of $a_{i,c,t}^I$	5
	g^I	Interbank interest rate intensity	0.1
Expectation Formation	θ^S	Error correction in calculation of $\mathrm{E}[\log(\omega_{s,t}^S)]$	0.1
	θ^Ω	Error correction in calculation of $\mathrm{E}[\log(\Omega_{c,t}^C)]$	0.01
	ψ^S	Memory parameter in calculation of $\mathrm{Var}(r_s^S)$, $\hat{\mathrm{Cov}}(r_{s1}^S, r_{s2}^S)$, $\hat{\mathrm{Cov}}(r_c^B, r^I)$, $\hat{\mathrm{Cov}}(r_s^S, r^I)$, $\hat{\mathrm{Cov}}(r_c^B, r_s^S)$	0.1
	ψ^B	Memory parameter in calculation of $\mathrm{Var}(r_c^B)$	0.01
Exogenous Agents			
Rest-of-world agent	λ_{row}	Vector of asset specific risk aversion parameters	Nb-assets 1-5: 50.000, 10.000 else
	E_{row}^{min}	Minimum equity	1000
	x^{row}	Aggressiveness factor	10
Central counterparty	x^R	Probability in calculation of haircuts and margin requirements	0.01
	r^M	Margin rate	0
	r^R	Repo rate	0
Investor deposits	m_D	Maturity of investor deposits	0.99
	α^D	Fraction of outstanding investor deposits at time of default	0.01
Lender of last resort	r^{CB}	Marginal Lending Rate	$\frac{0.05}{250}$
Market Maker	g^{MMS}	Price impact factor nb-securities	0.1
	g^{MMB}	Price impact factor bank bonds	0.1
	x^I	Stopping limit interbank	0.1
	x^{MMS}	Stopping limit nb-securities	0.1
	x^{MMB}	Stopping limit bank bonds	0.1

Table 5: Benchmark Simulation Parameters

B Commercial Bank Agents' Maturity Structure of Wholesale Debt

The mean-variance optimization problem of Section 3.2.1 is

$$\max_{\mathbf{a}} \underset{c,t}{\mathrm{E}}[S] - 0.5\lambda_{c,t} \underset{c,t}{\mathrm{Var}}(S), \tag{B.1}$$

with

$$\underset{c,t}{\mathrm{E}}[S] = \underset{c,t}{\mathrm{E}}[r^L] - \frac{W_{c,t}}{L_{c,t}} \left(a_{c,t} \underset{c,t}{\mathrm{E}}[\bar{r}^B] + (1-a_{c,t}) \underset{c,t}{\mathrm{E}}[\bar{r}^I]) \right) \tag{B.2}$$

$$\begin{aligned}
\underset{c,t}{\mathrm{Var}}(S) &= \underset{c,t}{\mathrm{Var}}(r^L) + (a_{c,t}\frac{W_{c,t}}{L_{c,t}})^2 \underset{c,t}{\mathrm{Var}}(\bar{r}^B) + ((1-a_{c,t})\frac{W_{c,t}}{L_{c,t}})^2 \underset{c,t}{\mathrm{Var}}(\bar{r}^I) && \text{(B.3)}\\
&+ \quad 2\left(-a_{c,t}\frac{W_{c,t}}{L_{c,t}} \underset{c,t}{\mathrm{Cov}}(r^L, \bar{r}^B) - (1-a_{c,t})\frac{W_{c,t}}{L_{c,t}} \underset{c,t}{\mathrm{Cov}}(r^L, \bar{r}^I) \right. && \text{(B.4)}\\
&+ \quad \left. a_{c,t}(1-a_{c,t})(\frac{W_{c,t}}{L_{c,t}})^2 \underset{c,t}{\mathrm{Cov}}(\bar{r}^B, \bar{r}^I) \right) && \text{(B.5)}
\end{aligned}$$

Differentiating Eq. (B.1) with respect to a yields

$$\begin{aligned}
\frac{\partial U_{c,t}}{\partial a_{c,t}} = &-\frac{W_{c,t}}{L_{c,t}} \left(\underset{c,t}{\mathrm{E}}[\bar{r}^B] - \underset{c,t}{\mathrm{E}}[\bar{r}^I] \right) - 0.5\lambda_c \left(2a_{c,t}(\frac{W_{c,t}}{L_{c,t}})^2 \underset{c,t}{\mathrm{Var}}(\bar{r}^B) - 2(1-a_{c,t})(\frac{W_{c,t}}{L_{c,t}})^2 \underset{c,t}{\mathrm{Var}}(\bar{r}^I) \right) \\
&-\lambda_c \left(-\frac{W_{c,t}}{L_{c,t}} \underset{c,t}{\mathrm{Cov}}(r^L, \bar{r}^B) + \frac{W_{c,t}}{L_{c,t}} \underset{c,t}{\mathrm{Cov}}(r^L, \bar{r}^I) + (1-2a_{c,t})(\frac{W_{c,t}}{L_{c,t}})^2 \underset{c,t}{\mathrm{Cov}}(\bar{r}^B, \bar{r}^I) \right)
\end{aligned} \tag{B.6}$$

After setting Eq. (B.6) to 0 and rearranging, one obtains

$$a^*_{c,t} = \frac{-\frac{L_{c,t}}{W_{c,t}} \frac{\mathrm{E}_{c,t}[\bar{r}^B] - \mathrm{E}_{c,t}[\bar{r}^I]}{\lambda_c} + \frac{L_{c,t}}{W_{c,t}} \left(\mathrm{Cov}_{c,t}(r^L, \bar{r}^B) - \mathrm{Cov}_{c,t}(r^L, \bar{r}^I) - \mathrm{Cov}_{c,t}(\bar{r}^B, \bar{r}^I) \right) + \mathrm{Var}_{c,t}(\bar{r}^I)}{\mathrm{Var}_{c,t}(\bar{r}^B) + \mathrm{Var}_{c,t}(\bar{r}^I) - 2\frac{L_{c,t}}{W_{c,t}} \mathrm{Cov}_{c,t}(\bar{r}^B, \bar{r}^I)} \tag{B.7}$$

Replacing $\mathrm{E}_{c,t}[\bar{r}^B]$ with $\mathrm{E}_{c,t}[m_B r^B_{c,t} + (1-m_B) r^B_{c,t+1}] = \mathrm{E}_{c,t}[r^B]$ and setting the covariance terms to zero yields

$$a^*_{c,t} = \frac{\mathrm{Var}_{c,t}(\bar{r}^I) - \frac{L_{c,t}}{W_{c,t}} \frac{\mathrm{E}_{c,t}[r^B] - \mathrm{E}_{c,t}[\bar{r}^I]}{\lambda_c}}{\mathrm{Var}_{c,t}(\bar{r}^I) + (1-m_B)^2 \mathrm{Var}_{c,t}(r^B)} \tag{B.8}$$

With

$$\begin{aligned}
\underset{c,t}{\mathrm{E}}[r^I] &= \underset{c,t}{\mathrm{E}}[r^I] = r^I_{c,t} && \text{(B.9)}\\
\underset{c,t}{\mathrm{E}}[\bar{r}^I] &= (1 - \underset{c,t}{\mathrm{E}}[a^{CB}, \psi^I]) \underset{c,t}{\mathrm{E}}[r^I] + \underset{c,t}{\mathrm{E}}[a^{CB}, \psi^I] r^{CB}_t && \text{(B.10)}\\
\underset{c,t}{\mathrm{E}}[a^{CB}_t] &= \underset{c,t}{\mathrm{E}}[a^{CB}] && \text{(B.11)}\\
\underset{c,t}{\mathrm{Var}}(x) &:= \hat{\mathrm{Var}}_{c,t}(x, \psi), && \text{(B.12)}
\end{aligned}$$

Eq. (B.8) can be rewritten as

$$a^*_{c,t} = \frac{\hat{\mathrm{Var}}_{c,t}(\bar{r}^I, \psi^I) - \frac{L_{c,t}}{W_{c,t}} \frac{\mathrm{E}_{c,t}[r^B] - \mathrm{E}_{c,t}[\bar{r}^I]}{\lambda_c}}{\hat{\mathrm{Var}}_{c,t}(\bar{r}^I, \psi^I) + (1-m_B)^2 \hat{\mathrm{Var}}_{c,t}(r^B, \psi^B)} \tag{B.13}$$

C Portfolio Optimization of Investment Bank Agents

We solve the mean-variance optimization problem

$$\mathbf{a}^*_{i,t} = \arg\max_{\mathbf{a}} \mathbf{a}'\mathrm{E}_{i,t}[\mathbf{r}] - 0.5\lambda_i \mathbf{a}'\mathbf{\Sigma}_{i,t}\mathbf{a} \qquad \text{s.t.} \tag{C.1}$$

$$a^R_{i,j,t} = \begin{cases} -(1-h^R_{j,t})a^S_{i,j,t} & \text{if } a^S_{i,j,t} \geq 0 \text{ and } h^R_{j,t} \leq h^D_t \\ 0 & \text{else} \end{cases} \tag{C.2}$$

$$a^M_{i,j,t} = \begin{cases} -(1+k_{j,t})a^S_{i,j,t} & \text{if } a^S_{i,j,t} < 0 \\ 0 & \text{else} \end{cases} \tag{C.3}$$

$$a^D_{i,t} = -(1-h^D_{j,t})(a^I_{i,t} + a^B_{i,t} + \sum_{j\in\mathcal{D}} a^S_{i,j,t}) \tag{C.4}$$

$$\{a^I_{i,t}, a^B_{i,t}, a^C_{i,t}\} \geq 0 \quad \text{and} \quad \mathbf{a}'\mathbf{1} = 1 \tag{C.5}$$

via an iterative process. In order to save computation time, we rewrite the problem given in Eq. (C.1)-(C.5) by integrating constraints (C.2)-(C.4) into the budget constraint and adjusting expected returns by associated financing costs. The rewritten problem only features nb-securities, the two types of interbank loans and cash, reducing the size of the weight vector from $3n^S+n^C+3$ to $n^S + n^C + 2$. We define $\hat{\mathbf{a}} = (\mathbf{a}^S, a^I, \mathbf{a}^B, a^C)'$ as the vector of asset weights and $\mathrm{E}_{i,t}[\hat{\mathbf{r}}] = (\hat{\mathbf{r}}^S_{i,t}, \hat{r}^I_{i,t}, \hat{\mathbf{r}}^B_{i,t}, r^C)'$ as the vector of adjusted returns. The individual components of the latter are

$$\hat{r}^S_{i,j,t} = \begin{cases} \mathrm{E}_{i,t}[r^S_j] - r^R_{j,t} & \text{if } (\mathrm{E}_{i,t}[r^S_j] \geq 0) \text{ and } h^R_{j,t} \leq h^D_{i,t} \\ \mathrm{E}_{i,t}[r^S_j] - r^D_{i,t} & \text{if } (\mathrm{E}_{i,t}[r^S_j] \geq 0) \text{ and } h^R_{j,t} > h^D_{i,t} \\ \mathrm{E}_{i,t}[r^S_j] - r^M_{j,t} & \text{if } (\mathrm{E}_{i,t}[r^S_j] < 0) \end{cases} \tag{C.6}$$

$$\hat{r}^I_{i,t} = \mathop{\mathrm{E}}_{i,t}[r^I] - r^D_{i,t} \tag{C.7}$$

$$\hat{r}^B_{i,c,t} = \mathop{\mathrm{E}}_{i,t}[r^B_c] - r^D_{i,t}, \tag{C.8}$$

The vector $\omega_{i,t} = (\omega^S, \omega^I, \omega^B, \omega^C)$ captures haircuts and margin requirements in the budget constraint. The individual components of $\omega_{i,t}$ are

$$\omega^S_{j,t} = \begin{cases} h^R_{j,t} & \text{if } \mathrm{E}_{i,t}[r^S_j] \geq 0 \text{ and } h^R_{j,t} \leq h^D_{i,t} \\ h^D_{i,t} & \text{if } \mathrm{E}_{i,t}[r^S_j] \geq 0 \text{ and } h^R_{j,t} > h^D_{i,t} \\ -k_{j,t} & \text{if } \mathrm{E}_{i,t}[r^S_j] < 0 \end{cases}, \tag{C.9}$$

$$\omega^I_{i,t} = h^D_{i,t} \tag{C.10}$$

$$\omega^B_{i,c,t} = h^D_{i,t} \tag{C.11}$$

$$\omega^C = 1, \tag{C.12}$$

$$\tag{C.13}$$

The problem can now be restated as

$$\mathbf{a}^*_{i,t} = \arg\max_{\mathbf{a}} \mathbf{a}'\mathrm{E}_{i,t}[\hat{\mathbf{r}}] - 0.5\lambda_i \mathbf{a}'\mathbf{\Sigma}_{i,t}\mathbf{a} \quad \text{s.t.} \quad \omega_{i,t}\mathbf{a} = 1. \tag{C.14}$$

The vector $\mathbf{a}^*_{i,t}$ is derived from the first order conditions in matrix form

$$\lambda_i \mathbf{\Sigma}\mathbf{a} + \mu\omega'_{i,t} = \hat{\mathbf{r}}_{i,t} \tag{C.15}$$

$$\omega_{i,t}\mathbf{a} = 1, \tag{C.16}$$

where μ is the Lagrange multiplier. With $\mathbf{V} := \begin{Bmatrix} \mathbf{\Sigma}_{i,t} & \omega'_{i,t} \\ \omega_{i,t} & 0 \end{Bmatrix}$, $\tilde{\mathbf{a}} := (\hat{\mathbf{a}}, \frac{\mu}{\lambda_i})'$ and $\mathbf{y} := (\frac{\hat{\mathbf{r}}_{i,t}}{\lambda_i}, 1)'$ and after rearranging, one obtains

$$\tilde{\mathbf{a}}^* = \mathbf{V}^{-1}\mathbf{y}. \tag{C.17}$$

The iterative process works as follows: We first solve for the weights vector $\tilde{\mathbf{a}}^*$ that maximizes the problem stated in Eq. (C.14). We then check whether the resulting weights violate any of following asset-specific conditions

$$a_j^S \geq 0 \text{ and } \underset{i,t}{\mathrm{E}}[r_j^S] \geq 0 \tag{C.18}$$

$$a_j^S < 0 \text{ and } \underset{i,t}{\mathrm{E}}[r_j^S] < 0 \tag{C.19}$$

$$a^I \geq 0 \tag{C.20}$$

$$a_c^B \geq 0 \tag{C.21}$$

$$a^C \geq 0 \tag{C.22}$$

$$\tag{C.23}$$

and set them to zero if this is the case. We repeat these two steps until the current weights vector equals that of the last iteration. Finally, to ensure optimality, we check whether the resulting vector satisfies the Kuhn-Tucker conditions.

D Supply of Investor Deposits

Starting from the law of motion for equity under the stress scenario, one arrives at a representation for $E_{i,t+\tau}$ that is dependent on the initial values for equity and deposits and the parameters $\rho_{i,t}^D$ and m_D:

$$E_{i,t+1} = (1+\rho_{i,t}^D)E_{i,t} + \rho_{i,t}^D D_{i,t} \tag{D.1}$$

$$E_{t+2} = (1+\rho_{i,t}^D)E_{i,t+1} + \rho_{i,t}^D D_{i,t+1} \tag{D.2}$$

$$= (1+\rho_{i,t}^D)((1+\rho_{i,t}^D)E_{i,t} + \rho_{i,t}^D D_{i,t}) + \rho_{i,t}^D m_D D_{i,t} \tag{D.3}$$

$$= (1+\rho_{i,t}^D)^2 E_{i,t} + (1+\rho_{i,t}^D)\rho_{i,t}^D D_{i,t} + \rho_{i,t}^D m_D D_{i,t} \tag{D.4}$$

$$E_{t+3} = (1+\rho_{i,t}^D)((1+\rho_{i,t}^D)^2 E_{i,t} + (1+\rho_{i,t}^D)\rho_{i,t}^D D_{i,t} + \rho_{i,t}^D m_D D_{i,t}) + \rho_{i,t}^D m_D^2 D_{i,t} \tag{D.5}$$

$$= (1+\rho_{i,t}^D)^3 E_{i,t} + (1+\rho_{i,t}^D)^2 \rho_{i,t}^D D_{i,t} + (1+\rho_{i,t}^D)\rho_{i,t}^D m_D D_{i,t} + \rho_{i,t}^D m_D^2 D_{i,t} \tag{D.6}$$

$$\ldots \tag{D.7}$$

$$E_{i,t+\tau} = (1+\rho_{i,t}^D)^\tau E_{i,t} + \rho_{i,t}^D D_{i,t} \sum_{x=0}^{\tau-1} (m_D)^x (1+\rho_{i,t}^D)^{\tau-1-x} \tag{D.8}$$

$$= (1+\rho_{i,t}^D)^\tau E_{i,t} + \rho_{i,t}^D D_{i,t} (1+\rho_{i,t}^D)^{\tau-1} \sum_{x=0}^{\tau-1} \left(\frac{m_D}{1+\rho_{i,t}^D} \right)^x \tag{D.9}$$

Given that $\frac{m_D}{1+\rho_{i,t}^D} \neq 1$, the geometric series in Eq. (D.9) can be rewritten and we obtain

$$E_{i,t+\tau} = (1+\rho_{i,t}^D)^\tau E_{i,t} + \rho_{i,t}^D D_{i,t} (1+\rho_{i,t}^D)^{\tau-1} \left(\frac{1 - \left(\frac{m_D}{1+\rho_{i,t}^D} \right)^\tau}{1 - \left(\frac{m_D}{1+\rho_{i,t}^D} \right)} \right). \tag{D.10}$$

If $\frac{m_D}{1+\rho^D_{i,t}} = 1$, Eq. (D.9) can be rewritten as

$$E_{i,t+\tau} = (1+\rho^D_{i,t})^\tau E_{i,t} + \rho^D_{i,t} D_{i,t} (1+\rho^D_{i,t})^{\tau-1}\tau \tag{D.11}$$

By replacing τ with T^{def}, setting $E_{i,t+T^{def}} = 0$ and rearranging, one arrives at the solution

$$D^*_{i,t} = \begin{cases} \frac{-(1+\rho^D_{i,t})E_t}{\rho^D_{i,t}\left(\frac{1-\left(\frac{m_D}{1+\rho^D_{i,t}}\right)^{T^{def}}}{1-\left(\frac{m_D}{1+\rho^D_{i,t}}\right)}\right)} & \text{if } \frac{m_D}{1+\rho^D_{i,t}} \neq 1 \\ \frac{-(1+\rho^D_{i,t})E_t}{\rho^D_{i,t} T^{Def}} & \text{if } \frac{m_D}{1+\rho^D_{i,t}} = 1 \end{cases} \tag{D.12}$$